TRANSACTIONS OF
THE
OPTICAL SOCIETY

VOL. XXXII. 1930-31 No. 3.

MODERN OPTICAL GLASS AS EXEMPLIFIED BY THE LIST OF THE PARSONS OPTICAL GLASS COMPANY DATED SEPTEMBER, 1926

BY T. SMITH, M.A., F.INST.P.
(Optics Department, The National Physical Laboratory.)

MS. received, 10th January, 1931. *Read and discussed, 12th February*, 1931.

ABSTRACT. The optical positions (μ, ν) of the glass types catalogued by the Parsons Optical Glass Company, and the relations between the dispersions for several segments of the visible spectrum, are exhibited graphically. It is shown that a knowledge of the refractive indices of any glass for three wave-lengths is sufficient to specify the index for the whole of the visible spectrum, and a knowledge of two indices is almost sufficient. In particular the partial dispersive ratios are almost functions (nearly linear functions) of ν only. This implies that there are no glasses suitable for making apochromatic telescope objectives of large relative aperture.

The graphs show that the general standard of accuracy of the measured indices is high. A new graphical method of interpolating refractive indices for glass is obtained.

THE attempts (which, successful and unsuccessful alike, go to make a fascinating story of scientific endeavour) made during the last hundred years to produce optical glass having novel combinations of properties have placed at the disposal of the optician of to-day a choice of materials which, both for variety and uniformity, would have amazed his predecessors. After the invention of the achromatic doublet the old optician for long had but two or three types of glass for his raw materials, and these were often so poor in quality that he had to select pieces here and there to obtain a reasonable approach to homogeneity. Nowadays glass which is not homogeneous is not supposed to be issued, and the list of a modern glass-works may contain particulars of from fifty to a hundred or more standard types. Though all of these are called for by one lens-maker or another, we may suspect that the manufacture of some could be discontinued without prejudice to the efficient production of any modern instrument. At the moment, however, the methods by which lenses are designed result in a demand for all these types. While still further types (not

mere variations) may yet be produced, it is not unlikely that the sign of healthy progress in the future will be a reduction in the number of varieties for which there is a regular call, rather than its enlargement, as in the past.

The characteristic feature of modern lists is the presence of glasses such as were first produced by the Schott glass-works at Jena. Not all of the varieties that have been issued by these works are now made. Indeed those glasses which at first attracted most attention, inasmuch as they departed most pronouncedly in desirable directions from the older kinds, are the very ones which disappeared earliest from subsequent catalogues.

The optical positions of the most important groups of glasses hitherto made are indicated in Fig. 1, which is taken from Glazebrook's *Dictionary of Applied Physics*. The most remarkable of the glasses described in the first Jena catalogue were the phosphate crowns, barium phosphate crowns, and borate flints. These are no longer made, as they have proved to be unstable. At a later date telescope crowns and telescope flints were introduced, with unusual relations between their relative dispersions and their values of ν, so that objectives partially or completely corrected for secondary spectrum could be constructed. Since then all of these glasses which departed to a notable extent from the normal position have been withdrawn. The abiding additions resulting from these investigations are the extended flint series, the borosilicate crowns, and the barium crowns, all of great industrial importance. The fluor crowns are comparatively recent glasses. Both they and the very dense flints are only likely to be employed when other glasses are useless, for they are abnormal in their physical properties and are very expensive.

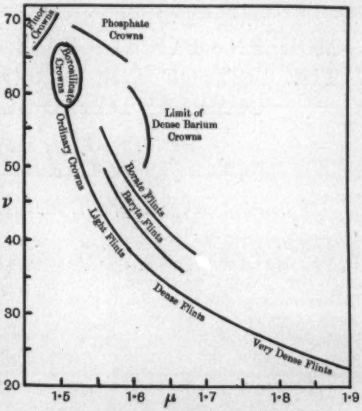


Fig. 1. The field of optical glasses.

It is noteworthy that all the new glasses lie to one side of the locus of the old glasses. For a given μ the new glasses have a higher ν, and for a given ν a higher μ. It would be of special interest to know whether this boundary can be crossed.

The 1926 list of the Parsons Optical Glass Company gives us a fair indication of modern commercial requirements. The optical positions of the various glasses are shown in Fig. 2. The glasses described in the catalogue as crowns are represented by light circles, and flints by dark circles. The three glasses shown by double circles are telescope flints, and may be expected to indicate how far a special type of abnormality is consistent with stability. Inspection of the figure suggests that an

attempt has been made to standardise selected refractive indices and also certain ν's. For example, there are three glasses with a ν of 53·7 having indices of approximately 1·55, 1·58 and 1·61. Again with an index of 1·613, apart from the flint glass having a ν of 36·9, we have dense barium crowns with ν's of 59·3, 57·6, 56·8, 55·8 and another glass slightly off the straight line with a ν of 58·5. Several other examples of the same kind of arrangement can be seen on inspecting the diagram. It is probably of distinct value to many designers to be able to choose a glass from such a series having only one degree of freedom, and a diagram is the best way of exhibiting this feature of the list. The designer of photographic lenses will not fail to notice that the refractive indices of the dense barium crowns have been made very high.

Fig. 2 differs from Fig. 1 in the scale for ν, the common logarithm instead of the number itself being taken as the variable. A chart plotted on this basis is much more useful than when ν itself is plotted on a uniform scale. The significant factor where two glasses are used in combination is the ratio of their ν's, and when the logarithmic scale is employed the separation of the representative points in the direction of the ν axis measures the factor required. These common logarithms are marked on the left side of the figure, and the corresponding ν scale is laid along the line for $\mu = 1{\cdot}67$. By the side of the ν scale is drawn another scale, marked κ, with a double set of figures. This should preferably be replaced by a separate strip that can be laid anywhere on the chart parallel to the direction shown. The lower end of the scale, marked $-\infty$ and ∞, is intended to be placed on the same ν level as one of a pair of glasses it is proposed to use in combination, and the reading corresponding to the ν of the other glass then gives the powers the two components must have to give an achromatic combination of unit power. It is of course assumed that the thicknesses and separations of the components may be neglected—an assumption very largely made in preliminary work. We notice for example that the component powers are always numerically greater than 2 and -1 unless the representative points are more widely separated in the ν direction than the flint glass $\mu = 1{\cdot}65$, $\nu = 33{\cdot}7$, and the fluor crown $\mu = 1{\cdot}49$, $\nu = 67{\cdot}9$. From the marked variation in the spacing along this scale we see that when the separation of the points is large the gain derived from any increase in separation is small. On the other hand variations from the standard position which may be insignificant with a large separation may involve a complete change of tools when the separation is much smaller. If any further commendation for charts of this kind be required, it may be pointed out that when used in combination with curves drawn on tracing-paper they enable glasses to be selected for satisfying assigned aberrational conditions. A discussion of the theory underlying this procedure will be found in a paper entitled* "Charts for assisting in the selection of suitable glasses for cemented doublets." It is worth observing that the method is much more widely applicable than this title indicates. It would be a great convenience to the user if glass-makers would include diagrams of this kind, all printed on agreed common scales for both μ and $\log \nu$, in their catalogues. The ruling should correspond to that of a standard squared paper, so

* *Trans. Opt. Soc.* **19** 1918 (169).

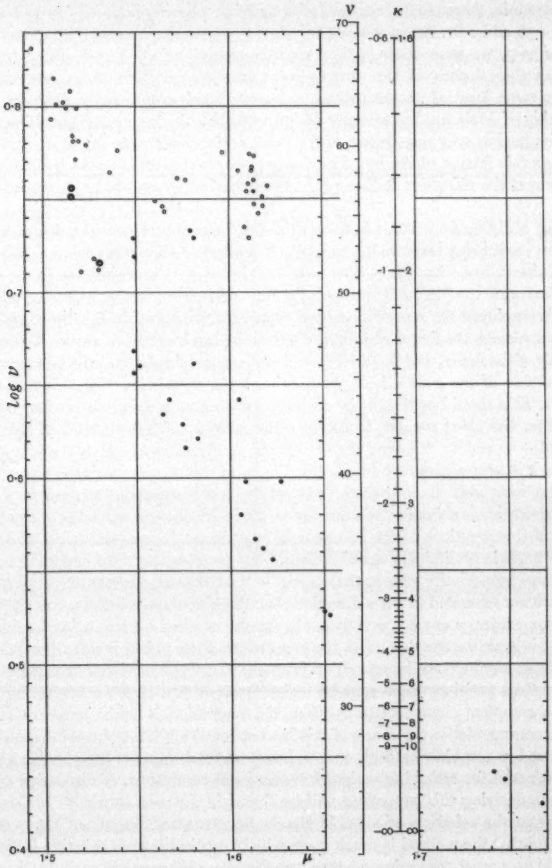


Fig. 2. Positions of Parsons Optical Glass Types.

that any worker could easily apply any curves of his own construction to the diagram.

The 1926 Parsons List differs from its predecessors in the spectral lines for which refractive indices are quoted. These are given in Table I.

Table I.

	Red		Yellow		Green	Blue		Violet	
Helium	*b*	—	—	*d*	—	—	—	—	—
Hydrogen	—	*C*	—	—	—	*F*	—	*G′*	—
Sodium	—	—	*D*	—	—	—	—	—	—
Mercury	—	—	—	—	*e*	—	*g*	—	*h*
Wave-length in μ	·706519	·656279 ± 6	·589295 ± 298	·587562	·546073	·486133	·435834	·434046	·404656

The notation employed for the spectral lines is not standard, but very convenient for this special purpose. It will be adopted in this paper. The values given here for the wave-lengths are taken from Landolt and Bornstein. Those given in the list contain only four figures—insufficient for many calculations—and in three cases the final figure is incorrect. The unit of the list is also given as $\mu\mu$ instead of $m\mu$. The indices for *D* and *G′* appear to have been given only for the convenience of those who wish to compare the properties of the glasses with those described in other lists. The substitution of the new yellow and violet lines is a step to be commended, and the additional lines which are included specify the refractive properties of the glasses throughout the visible spectrum with admirable completeness.

It is important to realise, more clearly than can be gained by inspection of the numbers in the table, how these spectral lines are related to one another. Fig. 3 has been drawn to represent this graphically. A number of equally spaced vertical lines are shown crossed by nine horizontal lines. Each of these horizontal lines is a scale on which the positions of the seven principal lines are shown. The scale on the lowest line is a uniform wave-length scale. The zero has been adjusted to represent line *C* and the interval *C*–*F* is chosen as the unit length. The middle horizontal line has the reciprocal of the wave-length as variable, and the points representing *C* and *F* are again brought into coincidence with the zero and unit points of the scale. In general if *w* is written for the reciprocal of the wave-length, so that *w* is the wave number, the various scales show w^n for values of *n* from − 1 to 3 at intervals of 0·5. The representative points of each line are joined by curves (which are vertical straight lines for *C* and *F*) and this enables other scales to be interpolated if desired. The scale marked 0 has log *w* for variable. The diagram shows clearly that our ideas of the relative separations of these lines depends very much on the scale we adopt. For example the separation between *F* and *h* is nearly equal to that between *C* and *F* if we use the w^3 scale, but is less than half that amount if we adopt the direct wave-length scale.

In the main part of the tables in the list the partial dispersions are given for *b*–*C*, *C*–*d*, *d*–*e*, *e*–*F*, *F*–*g*, *g*–*h*, and the partial ratios with respect to *C*–*F* are given for *b*–*d*, *C*–*e*, *e*–*F*, *F*–*g*, *g*–*h*. These ratios appear to have been selected because each

is approximately equal to $\frac{1}{2}$. The same space would have been better utilised if a complete set of ratios had been presented by relating d to either C or F and cutting out one of the two ratios involving e. For the purpose of this investigation all the partial ratios have been recalculated using the figures given for the partial dispersions. In some cases the final figure of the ratio given in the tables does not correspond, to the nearest final figure unit, to the values of the partial dispersions. In others the sum of C–d, d–e, e–F differed by a fifth-place unit from the quoted value of C–F. Corrected values have been used throughout this work. In the absence of sixth-place figures (which would be of the greatest value in further discussions on this subject) the fifth-place values given have been treated initially as though they were free from error.

The printed tables occupy six pages of the catalogue and specify 68 glasses. The first three pages include only crown glasses, and the last three only flints with the

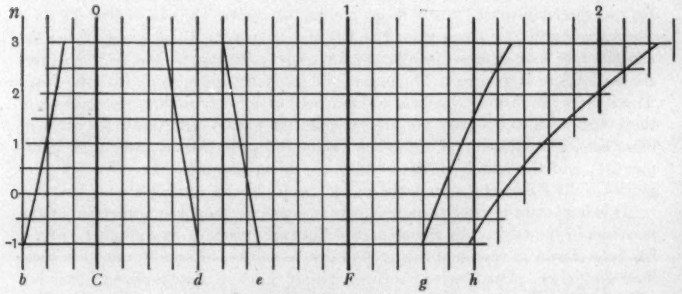


Fig. 3. Various spectral scales: $x = w^n = \lambda^{-n}$.

exception of three soft crown glasses. The first part of the examination of the dispersive properties consisted in plotting the various partial ratios against one another. The results are shown in Fig. 4. When the points representing the glasses on the first three pages had been laid down, it was apparent that they lay very closely on straight lines, which were then drawn on the diagram. Each crown glass is represented in the figure by a circle whose diameter corresponds to half a unit in the third decimal place of the partial ratio. With the crown glasses this corresponds approximately to five or less in the sixth decimal place of the refractive index. The points representing the glasses on the remaining three pages were then plotted, and the flint glasses represented by circles twice the size of those used for the crown (including soft crown) glasses. These points also lie well on straight lines, but the lines for the two halves of the list do not coincide. It is remarkable that each pair intersects in points for which the partial ratio $\frac{g-h}{C-F}$, which has been adopted as the abscissa, has the value 0·460. We may take these intersections as defining an ideal

glass which is a common member of the normal crown and the normal flint series, meaning by these terms glasses that have their representative points all lying on the

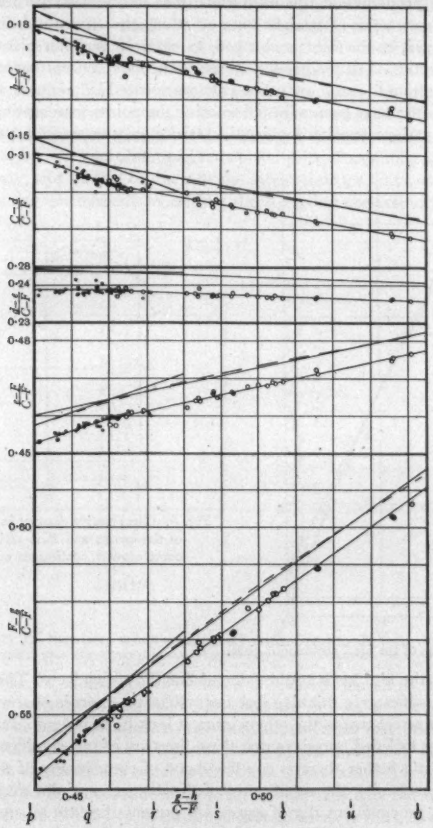


Fig. 4. Comparison of partial dispersions.

appropriate set of straight lines drawn in the diagram. Fig. 5 shows the variation of refractive index of this ideal glass plotted against w, the wave number. The curve

is clearly a smooth one. A straight line has been drawn through the *C* and *F* points to exhibit its curvature more clearly.

Given the properties of this ideal glass, it is only necessary to know how the slopes of the two series of straight lines on which the crown and flint glasses are represented vary, as the light is changed, to obtain a very fair picture of the refracting properties of all the glasses. These particulars are represented graphically in Fig. 6. The lines *C* and *F* are necessarily represented by points on the zero level, and the two scales have been adjusted to bring the points representing *h* into coincidence. The various remaining points lie satisfactorily on smooth curves. The full

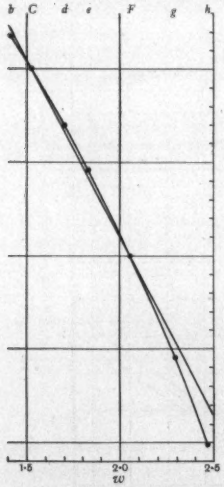


Fig. 5. Dispersion of the ideal crown-flint glass.

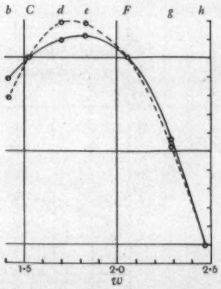


Fig. 6. Comparative changes in the dispersions of the crown and flint series: interrupted curve crowns, continuous curve flints.

line represents the flint locus and the dotted line the crown locus. The smoothness of the curves justifies us in thinking that any systematic errors in observations of any one spectrum line relative to the others must at least be of insignificant magnitude.

Having thus satisfied ourselves that these lines are of real significance, we proceed to consider whether there is any likelihood of error in any of the individual readings. Remembering the significance of the diameters of the circles drawn in Fig. 3, we can be confident that if errors are present they are all small. We can, however, be almost certain that some of the deviations of the observed points from the lines represent real errors. For example, there is a glass with the *g*–*h* ratio equal to 0·451, in which the *d*–*e* ratio is rather more than 0·002 high, and the *C*–*d* ratio almost 0·003 low, compared with the straight line values. The departures from

normal values in the other ratios do not exceed values which may be due to the limited number of figures used in the list. It appears very unlikely that the obvious explanation, viz. that the recorded reading for the *d* line should be increased by 2 in the fifth decimal place, is wrong. There are a few other cases where we can feel much confidence in amending the recorded figures. Our confidence is naturally less when the observed points lie on the intermediate lines and only depart from one or both of the extreme lines. Nevertheless we shall probably be correct here also in regarding any considerable departure from the line as an error. Thus we suggest that the index for line *b*, in the glass which has the *b–C* ratio equal to 0·178 and the *h–g* ratio 0·448, is too low by about two fifth-place units. Examining all the observations in this way, and leaving on one side the three telescope flints, we find that the table may require amendment by adding the following corrections in fifth-place units:

Table II.

Glass	Lines and corrections to indices
FC_1	$d, -1$
BSC_1	$g, -1$
BSC_2	$F, +1$
BSC_3	$g, +1$
BSC_7	$b, +2$
HC_1	$d, +2$
HC_5	$g, +1$
ZC_1	$e, -2$:$g, +1$
MBC_2	$d, +2$
SC_1	$d, -2$:$g, -1$
BLF_3	$b, -4$
ELF_1	$g, -1$
ELF_2	$b, +1$
ELF_3	$h, -1$
BF_1	$d, +1$
BF_3	$g, -1$
DF_4	$C, +1$
$DEDF_3$	$b, -4$

The brevity of this list, when compared with the number of observations, is itself a tribute to the accuracy of the measurements. The care which has been exercised will be the more apparent when it is realised that all the corrections of magnitude ± 1 may be correct from the point of view of this analysis without implying error in the observations or the calculations. They may be due to the expression of results in the form of differences when the number of figures used in computing each observation is suitably restricted. The remaining seven corrections are probably called for in consequence of real errors. It is only fair to remark that none of these errors is likely to be of importance to users of the glasses concerned.

Being now satisfied from this internal evidence that the accuracy of the data contained in the list is very high, we may look at the abnormal glasses. The three telescope flints are represented by circles of the size used for the other flint glasses, and they lie near to the intersection of the crown and flint lines, being situated

within the obtuse angles they form. They lie farther from some of the lines than most glasses: this may be a real departure, since they are intentionally abnormal glasses. On the other hand abnormality of this kind is not desired by the optician. He wishes the run of the dispersions to be exactly the same as that of the glass to be paired with it, and would like the abnormality to consist solely in a large ν difference between the two. While it must be admitted that we are here on much more uncertain ground than with the other glasses, I am inclined to think that there may be a small error in the figures for one at least of these glasses.

The other glasses which we may regard as somewhat abnormal are the densest of the flints. There is some indication of a systematic deviation from the straight line with all these dense glasses, and the explanation is probably that we should have drawn a slightly curved, instead of a straight, line as the locus. Obviously we should need strong additional evidence to justify us in adhering to our two straight lines rather than substituting a curve running smoothly from one direction to the other. How slight are the displacements involved in changing from the straight lines to the curve is shown above each locus. Above these straight lines, at a distance corresponding to 0·005 in the partial ratios, are drawn lines parallel to the actual tentative loci, but prolonged to show more clearly the angle at which they cut. Superposed on these straight lines, curves, which may well correspond to the real locus, have been represented by interrupted lines. No curve has been inserted in the *d–e* section as it would be impossible to distinguish it from the pair of straight lines.

To sum up, we have reached the conclusion that modern commercial optical glasses form a single series as regards their relative partial dispersions, the knowledge of any one of these ratios enabling all other ratios to be found without ambiguity. The relation between two such ratios is almost, but not quite, linear.

This definite relation between the various ratios is of the highest importance. It means that an accurate knowledge of the refractive indices of a glass for three suitably separated spectrum lines is sufficient to give the value of the index for any assigned line in the visible spectrum. For example, if we know *C*, *F*, *h*, we can calculate the partial ratio $\frac{\mu_F - \mu_h}{\mu_C - \mu_F}$, and by the relations between partial ratios find say $\frac{\mu_F - \mu_g}{\mu_C - \mu_F}$, and thence μ_g, or $\frac{\mu_e - \mu_F}{\mu_C - \mu_F}$ and thus μ_e. Clearly the process is quite general. We may state this conclusion in the form that a single partial ratio gives all partial ratios. Since the figures on which this conclusion rests relate to many spectral lines and have passed successfully the most stringent examination to which we can reasonably subject them, and since they are representative of all kinds of glass, we may confidently base further investigations on this conclusion and on the numerical data of the catalogue.

We now know that the knowledge of the refractive indices for three lines is sufficient to specify the dispersion over the whole visible range, and it is natural to enquire whether a knowledge of three indices is necessary. The meaning of Figs. 1 and 2 is clearly that a knowledge of the index for one line is inadequate, and we therefore only have to consider whether it is sufficient to know the index for two lines.

For the construction of such a relation only two independent variables commend themselves for consideration, viz. a particular dispersion such as $\mu_F - \mu_C$, and ν,

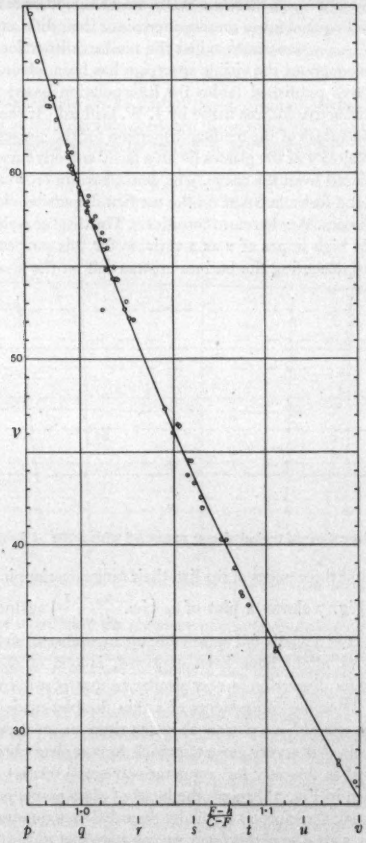


Fig. 7. Relation between ν and a partial dispersive ratio.

sometimes called the constringence. The usual definition of ν involves the indices for three lines, but it is easy to modify the definition so that only two are involved.

For instance we may define it as $(\mu_F - 1)/(\mu_F - \mu_C)$ or as $(\mu_C - 1)/(\mu_F - \mu_C)$, values which differ by unity. If any importance is attached to a symmetrical form we may use $(\mu_F + \mu_C - 2)/2\,(\mu_F - \mu_C)$, which is the mean of the other two expressions. As variables they are all equivalent to one another, since their differences are constants.

The use of $\mu_F - \mu_C$, sometimes called the median dispersion, as the basis for index differences throughout the visible spectrum has been advocated by J. W. and E. Gifford, who have published tables for interpolation based on the six-figure measurements of refractive indices made by J. W. Gifford. If the F–h partial ratio is plotted against the values of the median dispersion for the glasses of this catalogue, it is seen that the majority of the glasses lie on a fairly smooth curve, but the barium glasses are all displaced from the curve. The dense barium crowns in particular are displaced so far that a formula based on the median dispersion alone would be unsuitable for some glasses. We therefore consider ν. The diagrams already constructed lead us to entertain high hopes of ν as a variable for this purpose, for in plotting Fig. 4 the points representing the barium crowns fell on the short line drawn for

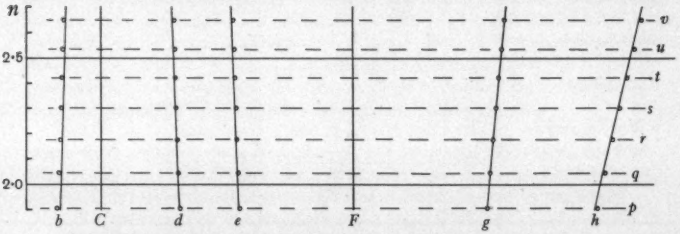


Fig. 8. Dispersions of typical glasses compared with scales of form $x = w^n$.

the glasses on the first three pages of the list, their range overlapping that of the hard crowns slightly. Fig. 7 shows a plot of $\nu_F \left(\text{i.e. } \frac{\mu_F - 1}{\mu_F - \mu_C}\right)$ against $\frac{\mu_h - \mu_F}{\mu_F - \mu_C}$. The representative points as a whole fall reasonably on an almost straight line. The most outstanding glass, as might have been expected, is one of the telescope flints (TF 3). The run of its dispersion is very similar to that of a normal glass with a ν four units higher. Thus the components of a thin doublet made apochromatic by the use of this glass would have powers about 13 times as great as that of the combination. This means that a very close approach to complete chromatic correction can only be achieved in this way for comparatively small relative apertures.

The curve drawn in Fig. 7 through the locus of glass points is a parabola. This partial dispersion may therefore as a rule be regarded as a quadratic function of ν. As the curves of Fig. 3 are also of this form, we can state that all the partial dispersions are approximately quadratic functions of ν.

If we compare the numerical values of the partial dispersive ratios recorded in Fig. 3 with the relative lengths into which the various horizontal lines of Fig. 2 are

cut by the curves representing the corresponding spectral lines, we can hardly fail to be struck by the closeness of the correspondence we observe. The indices of w involved (the figures with which the horizontal lines of Fig. 2 are labelled) are chiefly those between 1·5 and 2·5, the former corresponding to the glasses with a high ν, the latter to those with a low ν. It follows that a dispersive formula of the type

$$\mu = a + bw^n,$$

where n is a function of one of the partial ratios, or alternatively is a function of ν, will be susceptible of considerable accuracy throughout the whole visible spectrum. The appropriate value of n may be found in any particular case at once from a relation of the type

$$\frac{\mu_1 - \mu_2}{\mu_2 - \mu_3} = \frac{w_1^n - w_2^n}{w_2^n - w_3^n}.$$

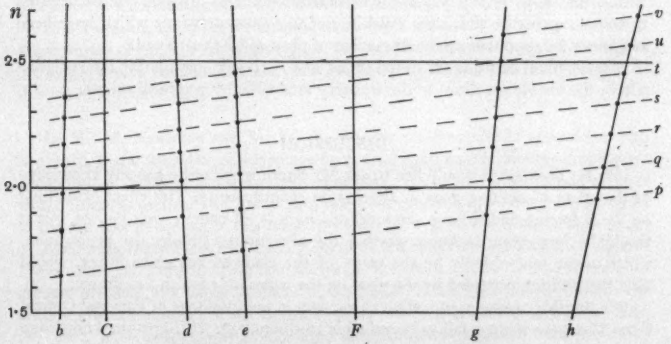


Fig. 9. Dispersions of typical glasses shown on tilted scales.

The readiest way of examining the accuracy of this formula is to plot on transparent squared paper sets of partial dispersions represented by the curves of Fig. 4, taking the same unit for the *C*–*F* interval as was adopted in constructing Fig. 3. We then have on the transparent paper a series of points lying on a straight line representing the dispersive properties of a glass for the same intervals as are marked off on any horizontal line of Fig. 3 by the transverse curves. The transparent paper should be laid on Fig. 3 with the straight line representing any particular glass horizontal and the points representing *C* and *F* on the zero and unit lines of Fig. 3. The paper is adjusted by vertical movements until the remaining points lie as close as possible to the corresponding curves. Fig. 8 shows some characteristic results. The letters *p*, *q*, *r*, *s*, *t*, *u*, *v* denote ideal glasses with values of $\frac{\mu_h - \mu_F}{\mu_F - \mu_C}$ of 0·9700, 1·0000, 1·0300, 1·0600, 1·0900, 1·1200 and 1·1500 respectively. This series may be accepted as a satisfactory representation of the whole range of commercial glasses. It will be

seen that the fit cannot be made exact for all the points simultaneously. A fit correct to about the accuracy we can obtain from graphs on squared paper may however be obtained by a slight rotation of the transparent paper, so that the end of the glass line for the violet end of the spectrum lies on a part of the chart corresponding to a higher n than that for the red end. Fig. 9 represents the superposition attained in this way for the ideal glasses already defined. For clearness the scale for n in Figs. 8 and 9 has been increased fivefold compared with Fig. 3. The plot shows that optical glasses in general are members of an orderly family, and justifies us in expecting that we can find a comparatively simple formula which will express the dispersive properties of all its members. A promising method of reaching such a formula would be to try the effect of modifying the expression we have been considering in various ways. Since the construction of dispersive formulae hardly lies within the scope of this paper, the consideration of this problem will be deferred to another paper in which the reliability of various expressions which have been suggested for representing the dispersion of glass will be examined.

The method of using the tilted index line is clearly suitable for interpolation within the visible spectrum to the accuracy attainable by graphical means.

DISCUSSION

Mr F. Twyman: I would like to ask Mr Smith whether he has any knowledge of the effect of chilling glass in altering its characteristics. The effect of chilling on the refractive index for a single wave-length is, of course, well known, but it would be interesting to know whether the alterations of molecular aggregation, which must undoubtedly be the cause of the changed refractive index, would alter the position occupied by the glass on the curves Mr Smith has drawn.

Mr Smith's careful examination of the data in the catalogue of Parsons' Optical Glass Company cannot fail to be valuable to computers. Hitherto the knowledge required by a computer in respect of any glass he intended to use would only be fully provided by a large number of readings taken with great care, and only to be obtained with the desired accuracy on very powerful and accurate instruments. It would seem to be the case that computers could now derive a like amount of knowledge about any particular glass of certain of the types covered by the catalogue in question, provided a measurement of one refractive index had been made with a suitable accuracy.

This, however, still leaves the opticians with the practical difficulties which arise from variations of refractive index in blocks or mouldings all made from the same melting. It would be interesting to know if Mr Smith has any recent information on the degree of uniformity in refractive index throughout meltings which is being obtained to-day by the various glass manufacturers.

The thanks of all opticians are due to Mr Smith for the most laborious but exceedingly valuable examination of these figures.

Mr J. W. Perry: Mr Smith has in my opinion rendered to all who have need of accurate particulars of optical glass an important service by conducting this

critical examination of Messrs Parsons' glass list and by the generalisations which he has here established. Glass lists contain a large amount of information which, however, is presented in a form frequently necessitating a large amount of work to be done on it in order to render it suitable for the intended applications, quite apart from tests of internal accuracy. It is gratifying to learn of the high order of accuracy of the listed constants. The specification of dispersion data through the coefficients of a dispersion formula of generally acceptable type such as Mr Smith has laid foundation for would greatly curtail the work otherwise necessary in transforming the optical data and would I think be accurate enough for most work.

It may, however, be added that from the practical point of view it is very desirable to include in new glass lists beside the constants as expressed in the more modern notation also a characteristic constant such as $(G' - F)/(F - C)$ which will serve as a connecting link with older publications dealing with optical glass.

It is a notable achievement to show the twofold uniformity which exists within the diversity of present-day glass types. This achievement, however, also prompts the enquiry as to whether artificial glass generally regarded is inherently subject to this irksome restriction.

Dr W. M. Hampton and Mr W. N. Wheat: We are quite in agreement with Mr Smith that it would be an advantage for optical glass manufacturers to reproduce a standard μ and log ν curve in their catalogues. We ourselves published a graph with a linear scale for ν in Chance Brothers' catalogue for 1926, but having regard to the remarks in the paper this might preferably be altered to a logarithmic scale.

Mr Smith reproduces the μ and ν curve in Glazebrook's *Dictionary of Applied Physics* with the same limiting boundaries for the various types. These boundaries are now obsolete, as Chance Brothers' dense barium crown type 8065 is outside the barium crown boundary of the diagram, while a new glass, a dense barium flint, No. 5902, $\mu_D = 1{\cdot}6437$, $\nu = 48{\cdot}3$, is even more extreme compared with the limits of this diagram.

We are not convinced that there is sufficient justification for deducing two straight lines from the points for the relative partial dispersions and regard the claim that they all intersect at the value $(g - h)/(C - F) = 0{\cdot}460$ as particularly untrustworthy. The broken curves in Fig. 4 seem to join the points more accurately.

For estimating approximately the partial dispersions of a glass, we have from time to time employed curves showing how these vary with the mean dispersion $\mu_F - \mu_C$. If the glasses are sorted into groups having similar compositions and the values plotted in this way, it will be found that the order of accuracy of a curve joining the points is very high indeed. This, we suggest, is a better way of representing the facts than dividing the catalogue types arbitrarily into two classes—crowns and flints—with telescope flints as a curious departure from the remainder.

Considering the question of optical glass manufacture from the point of view of reducing costs, we heartily concur with Mr Smith's suggestion that the number of types should be reduced, since this can clearly be done without sacrificing efficiency.

Mr Smith: I have no knowledge of the degree of uniformity found in refractive index throughout meltings other than that which has been given in a few scientific papers. We should all like to know much more on this subject, and as much precise information is doubtless in the possession of firms in the glass and optical industries it would be of general service if it were published. A joint paper summarising the facts known in these two industries would be of the first importance.

For ordinary purposes it would appear to be sufficient, for each glass melting, to take careful readings of refractive index on two lines—preferably widely separated, say *C* and *h* or *C* and *g*. For systems to be corrected to a higher order the indices for three lines, say *C*, *F*, *h*, should be ample. The time so saved would enable more attention to be given to the variations of index present in the meltings, and to special problems such as the refractive properties for a large number of lines of glasses which are intentionally abnormal. The detailed examination of abnormal glasses will yield a definite answer to Mr Perry's question whether the twofold restriction inevitably accompanies stability in glasses.

I have no information on the effects of chilling glass to which Mr Twyman referred.

I am indebted to Dr Hampton and Mr Wheat for calling attention to the two glasses of Messrs Chance Bros. which lie outside the limits of Fig. 1. This diagram, which is only here intended to give a rough indication of the ground covered by glasses, exhibits the position reached about the time of the war. The production of glasses outside these limits, with good optical, chemical, and mechanical properties, is clearly of much importance to the optician.

The two sets of lines Dr Hampton and Mr Wheat criticise were drawn independently of one another as curves of best fit when the division of the list by pages had been made. The nature of the separating factor should indicate that two sets of straight lines are likely to be less significant than a single set of curves, and this view is expressed in the paper.

I am particularly interested in the statement about the curves drawn through points representing the various partial ratios of glasses of similar composition. The makers of glasses are apparently in possession of unpublished information which is of interest to users. It would be a distinct service if these curves (presumably giving fairly conclusive evidence of a lateral shift of loci such as those of Figs. 4 and 7 for glasses of different groups) were published. The glass-maker alone is in a position to effect the grouping of the glasses and present the results in the most conclusive form. This information would go a long way to answering Mr Twyman's and Mr Perry's questions, and would be of the first importance in a critical examination of formulae for representing the dispersive properties of glass. I hope it will be possible for Messrs Chance Bros. to add to the services they have already rendered to the optical industry by presenting it with this important information.

FUSED BIFOCAL SPECTACLE LENSES

By A. WHITWELL, M.A.

MS. received, 20th February, 1931. Read and discussed, 26th March, 1931.

ABSTRACT. In designing a fused bifocal spectacle lens we have at our disposal three radii which can be varied to satisfy the two conditions, that the distance power and the reading addition have given values, so leaving one degree of freedom.

An expression is found for the convergence of the circles of least confusion of oblique pencils of small aperture; from this, by equating the aberration term to zero, we obtain a quadratic equation the roots of which fix the radius of one surface of the lens subject to the third condition that the circles of least confusion lie on a sphere and so focus on the macula of the eye.

In making fused bifocal spectacle lenses it is usual first to grind a spherical depression in a flat blank of crown glass; in this depression is fitted a plano-convex lens or button of flint glass of the same curvature as the depression; the two parts are then fused together and finally two spherical surfaces are ground on the two faces of the blank (see Fig. 1).

The middle part of this lens then forms the reading segment having three surfaces of radii r_1, r_2, r_3 and two glasses of indices μ_1 and μ_2; the outer part forms the distance lens having the two surfaces r_1 and r_3 and glass of index μ_1.

In some cases the button itself is made by grinding a spherical surface on a composite flat blank made by fusing together edge to edge two plates, one of flint and the other of crown glass, as shown in Fig. 1.

Powers of distance and reading lenses

Referring to Fig. 1 and calling the power of the distance lens D and that of the reading segment $D + R$, where R is the reading addition in dioptres, we have

$$D = (\mu_1 - 1)(\rho_1 - \rho_3) \qquad \text{.....(1),}$$

$$D + R = (\mu_1 - 1)\rho_1 + (\mu_2 - \mu_1)\rho_2 + (1 - \mu_2)\rho_3 \qquad \text{.....(2),}$$

where $\rho_1 = 1/r_1, \quad \rho_2 = 1/r_2, \quad \rho_3 = 1/r_3.$

Equation (2) may be written

$$D + R = (\mu_1 - 1)(\rho_1 - \rho_2) + (\mu_2 - 1)(\rho_2 - \rho_3) \qquad \text{.....(3).}$$

In equation (2) the three terms on the right are the powers of the three surfaces and in (3) the two terms on the right are the powers of the two separate lenses in contact.

Subtracting (1) from (2) we find

$$R = (\mu_2 - \mu_1)(\rho_2 - \rho_3) \qquad \text{.....(4).}$$

By the use of equations (1) and (4) we can write ρ_1 and ρ_2 in terms of ρ_3 as follows:

$$\rho_1 = \frac{D}{\mu_1 - 1} + \rho_3 \qquad \text{.....(5),}$$

$$\rho_2 = \frac{R}{\mu_2 - \mu_1} + \rho_3 \qquad \text{.....(6).}$$

Equations (1) and (2) show that in designing a fused bifocal lens we have at our disposal five quantities—three radii and two indices of refraction.

The glasses are usually chosen to minimize as far as possible the chromatic aberration of the reading segment and we are left with three radii which can be varied to satisfy two conditions, viz. that the power of the distance lens must be D and the reading addition R dioptres.

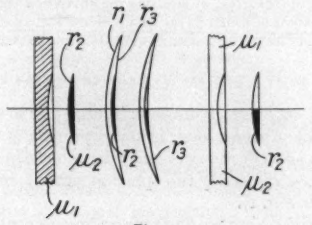


Fig. 1.

$$D=(\mu_1-1)(\rho_1-\rho_3), \quad D+R=(\mu_1-1)\rho_1+(\mu_2-\mu_1)\rho_2+(1-\mu_2)\rho_3,$$

$$R=(\mu_2-\mu_1)(\rho_2-\rho_3), \quad \rho_1=\frac{D}{\mu_1-1}+\rho_3, \quad \rho_2=\frac{R}{\mu_2-\mu_1}+\rho_3.$$

We have thus one degree of freedom, and any number of lenses can be designed to satisfy the two conditions when paraxial rays only are taken into account.

The one degree of freedom may be used in making the lens give the best possible image of objects off the axis when the chief rays of the imaging pencils make small angles with the optic axis.

Fig. 2 shows an axial section of a spectacle lens and an eye. The centre of rotation of the eye is marked d and is supposed to be at a distance of d metres behind the last vertex of the lens. A small object on the axis at a_1 is at a distance of $-a_1$ metres from the first vertex of the lens. The point a_2' at a distance of a_2' metres from the last vertex and at a distance of $a_2' - d$ metres from the centre of rotation of the eye is conjugate with the point a_1 with respect to the lens. The optical system of the eye will be supposed to focus the image at a_2' on the macula at m.

Now as the eye rotates on its centre of rotation to view points on the object off the axis the point a_2' must lie on a sphere of radius $a_2' - d$ and centre d if the images are to focus on the macula without any change of accommodation, since the chief rays of all imaging pencils must pass through the centre of rotation of the eye.

When the accommodation of the eye is completely relaxed, the point a_2' is called

the far point and the sphere is called the far-point sphere. For a near object I shall call the sphere, for convenience of reference, the conjugate image sphere.

The oblique pencils in general suffer from coma and astigmatism. I shall consider the coma negligible and suppose the pencils to be pure astigmatic pencils. In general neither the primary nor the secondary focus will lie on the conjugate image sphere. It is possible to use our remaining degree of freedom in choosing the curvatures of the lens so that either the primary or the secondary focus will lie on the conjugate image sphere but not both.

Alternatively we may make the primary and secondary foci coincide, that is, we can correct the astigmatism.

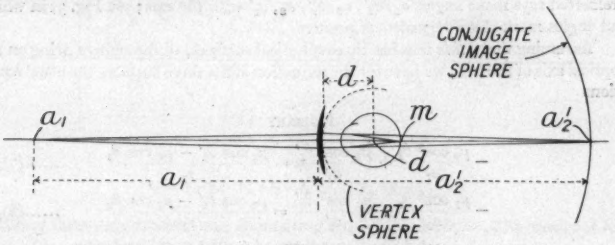


Fig. 2.

In this case the coincident foci will not in general lie on the conjugate image sphere except at the axial point, and consequently the images of objects off the axis will not focus on the macula.

If one section of a pure astigmatic pencil, formed by a circular stop such as the pupil of the eye, be circular, it may be shown that there will be another circular section commonly called the circle of least confusion.

If p, s, and c be the distances of the primary and secondary foci and the circle of least confusion from this circular aperture, measured along the chief ray, then we have

$$\frac{1}{p}+\frac{1}{s}=\frac{2}{c},$$

or the convergence of the circle of least confusion is the arithmetic mean of the convergences of the primary and secondary foci.

It is possible to use the available degree of freedom in selecting the curvatures of the three surfaces of the lens in such a way that the circles of least confusion of oblique pencils of small aperture shall all lie on the conjugate image sphere, and I propose in what follows to find an expression for the reciprocal of the distance of the circle of least confusion from a circular aperture.

From this expression by equating the aberration term to zero we get an equation, independent of the aperture, which will enable us to calculate the radii of the three surfaces of the lens.

Notation

I shall assume that light travels from left to right, that the radius of a surface is positive when the surface is convex to the incident light, and that distances measured from a point of incidence along a ray in the direction of light are positive.

These distances will be represented by the letter a with dashes and subscript numerals to denote the surface; thus a_1, a_1' represent distances of object and image from the first surface; a_2, a_2' from the second and a_3, a_3' from the third. Angles of incidence and refraction will be denoted by $\theta_1, \theta_1', \theta_2, \theta_2', \theta_3, \theta_3'$; indices of refraction by $\mu_0, \mu_1, \mu_2, \mu_3$; normals make angles $\beta_1, \beta_2, \beta_3$ with the optical axis, and incident and refracted rays make angles $\alpha_1, \alpha_1', \alpha_2, \alpha_2', \alpha_3, \alpha_3'$ with the axis; see Fig. 3, in which all angles marked are regarded as positive.

Assuming that a *thin* lens has three spherical surfaces, all the centres being on the optical axis of the lens, we have for the refraction at the three surfaces the usual equations

Primary

$$-\frac{\mu_0 \cos^2\theta_1}{a_1} + \frac{\mu_1 \cos^2\theta_1'}{a_1'} = \frac{\mu_1 \cos\theta_1' - \mu_0 \cos\theta_1}{r_1} \qquad \text{.....(7),}$$

$$-\frac{\mu_1 \cos^2\theta_2}{a_2} + \frac{\mu_2 \cos^2\theta_2'}{a_2'} = \frac{\mu_2 \cos\theta_2' - \mu_1 \cos\theta_2}{r_2} \qquad \text{.....(8),}$$

$$-\frac{\mu_2 \cos^2\theta_3}{a_3} + \frac{\mu_3 \cos^2\theta_3'}{a_3'} = \frac{\mu_3 \cos\theta_3' - \mu_2 \cos\theta_3}{r_3} \qquad \text{.....(9).}$$

On multiplying these three equations by suitable factors, adding them together, and putting $\mu_0 = \mu_3 = 1$, we find

$$\begin{aligned}\frac{1}{a_3'} = \frac{1}{a_1}\left(\frac{\cos^2\theta_1 \cos^2\theta_2 \cos^2\theta_3}{\cos^2\theta_1' \cos^2\theta_2' \cos^2\theta_3'}\right) &+ \left(\frac{\mu_1 \cos\theta_1' - \cos\theta_1}{r_1}\right)\left(\frac{\cos^2\theta_2 \cos^2\theta_3}{\cos^2\theta_1' \cos^2\theta_2' \cos^2\theta_3'}\right) \\ &+ \left(\frac{\mu_2 \cos\theta_2' - \mu_1 \cos\theta_2}{r_2}\right)\left(\frac{\cos^2\theta_3}{\cos^2\theta_2' \cos^2\theta_3'}\right) \\ &+ \left(\frac{\cos\theta_3' - \mu_2 \cos\theta_3}{r_3}\right)\left(\frac{1}{\cos^2\theta_3'}\right) \qquad \text{.....(10).}\end{aligned}$$

Secondary

$$-\frac{\mu_0}{a_1} + \frac{\mu_1}{a_1'} = \frac{\mu_1 \cos\theta_1' - \mu_0 \cos\theta_1}{r_1} \qquad \text{.....(11),}$$

$$-\frac{\mu_1}{a_2} + \frac{\mu_2}{a_2'} = \frac{\mu_2 \cos\theta_2' - \mu_1 \cos\theta_2}{r_2} \qquad \text{.....(12),}$$

$$-\frac{\mu_2}{a_3} + \frac{\mu_3}{a_3'} = \frac{\mu_3 \cos\theta_3' - \mu_2 \cos\theta_3}{r_3} \qquad \text{.....(13),}$$

$$\frac{1}{a_3'} = \frac{1}{a_1} + \frac{\mu_1 \cos\theta_1' - \cos\theta_1}{r_1} + \frac{\mu_2 \cos\theta_2' - \mu_1 \cos\theta_2}{r_2} + \frac{\cos\theta_3' - \mu_2 \cos\theta_3}{r_3} \qquad \text{.....(14).}$$

We can simplify equations (10) and (14) by approximating in the usual manner. We may substitute for the cosines, $\cos^2\theta_1 = 1 - \sin^2\theta_1$; $\cos\theta_1 = 1 - \sin^2\theta_1/2$, etc., neglecting all terms that contain higher powers of the sines than the second. The law of refraction gives us

$$\sin\theta_1 = \mu_1 \sin\theta_1'; \quad \mu_1 \sin\theta_2 = \mu_2 \sin\theta_2'; \quad \mu_2 \sin\theta_3 = \sin\theta_3'.$$

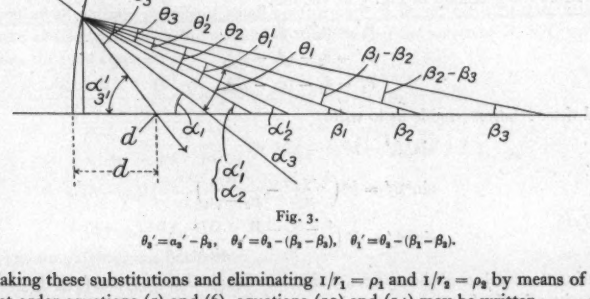


Fig. 3.

$\theta_3' = \alpha_3' - \beta_3$, $\theta_2' = \theta_3 - (\beta_2 - \beta_3)$, $\theta_1' = \theta_2 - (\beta_1 - \beta_2)$.

Making these substitutions and eliminating $1/r_1 = \rho_1$ and $1/r_2 = \rho_2$ by means of the first order equations (5) and (6), equations (10) and (14) may be written

PRIMARY

$$\frac{1}{a_3'} = \frac{1}{a_1} + D + R + \sin^2\theta_1' \left[\left(\frac{\mu_1 + 2}{2} \right) D + \frac{(\mu_1 + 2)(\mu_1 - 1)}{2} \rho_2 - \frac{1}{a_1}(\mu_1^2 - 1) \right]$$
$$+ \sin^2\theta_2' \left[-\left(\frac{\mu_2^2 - \mu_1^2}{\mu_1^2} \right) D + \left(\frac{\mu_2 + 2\mu_1}{2\mu_1} \right) R + \left(\frac{\mu_2^2 - \mu_1^2}{\mu_1^2} - \frac{\mu_2(\mu_2 - \mu_1)}{2\mu_1} \right) \rho_2 \right.$$
$$\left. - \frac{1}{a_1} \left(\frac{\mu_2^2 - \mu_1^2}{\mu_1^2} \right) \right]$$
$$+ \sin^2\theta_3' \left[\frac{\mu_2^2 - 1}{\mu_2^2} (D + R + 1/a_1) - \left(\frac{\mu_2^2 - 1}{\mu_2^2} + \frac{1 - \mu_2}{2\mu_2} \right) \rho_2 \right] \quad \text{.....(15).}$$

SECONDARY

$$\frac{1}{a_3'} = \frac{1}{a_1} + D + R + \sin^2\theta_1' \left[\frac{\mu_2 D}{2} + \frac{\mu_1(\mu_1 - 1)\rho_2}{2} \right]$$
$$+ \sin^2\theta_2' \left[\frac{\mu_2 R}{2\mu_1} + \frac{\mu_2(\mu_2 - \mu_1)\rho_2}{2\mu_1} \right]$$
$$+ \sin^2\theta_3' \left[\frac{(1 - \mu_2)\rho_2}{2\mu_2} \right] \quad \text{.....(16).}$$

We must now find the values of $\sin^2\theta_1'$, $\sin^2\theta_2'$ and $\sin^2\theta_3'$ in terms of ρ_2, D, R, δ, μ_1, μ_2 and h. The letters undefined are δ and h, and $\delta = 1/d$ = the reciprocal of

the distance of the centre of rotation from the last vertex and h = the height above the optical axis of the point of emergence from the lens of the chief ray of any oblique pencil which of course passes through the centre of rotation.

Since the sines only occur as squares we can, to our order of approximation, write

$$\sin \alpha_3' = h\delta; \quad \sin \beta_3 = h\rho_3,$$
$$\sin \beta_1 = h\rho_1; \quad \sin \beta_3 = h\rho_3.$$

From the geometry of Fig. 3 we see that

$$\theta_3' = \alpha_3' - \beta_3,$$
$$\theta_2' = \theta_3 - (\beta_2 - \beta_3),$$
$$\theta_1' = \theta_2 - (\beta_1 - \beta_2),$$

and these relations enable us to write

$$\sin^2 \theta_3' = h^2 (\delta - \rho_3)^2,$$
$$\sin^2 \theta_2' = h^2 \left(\frac{\delta - \rho_3}{\mu_2} - \frac{R}{\mu_2 - \mu_1}\right)^2,$$
$$\sin^2 \theta_1' = h^2 \left(\frac{\delta - \rho_3 - R}{\mu_1} - \frac{D}{\mu_1 - 1}\right)^2.$$

Putting these values of the squares of the sines in equations (15) and (16), they may be written in the forms

$$\text{Primary} \qquad \frac{1}{a_3'} = \frac{1}{a_1} + D + R + h^2 P \qquad \ldots\ldots(17).$$

$$\text{Secondary} \qquad \frac{1}{a_3'} = \frac{1}{a_1} + D + R + h^2 S \qquad \ldots\ldots(18).$$

The convergences $1/a_3'$ in the above equations are measured from the point of emergence of the chief ray of any oblique pencil from the third surface of the lens.

We require their values measured from a circular section of the pencil. Instead of taking the entrance pupil of the eye it is permissible to take for the aperture a small circle on a sphere centered at the centre of rotation and passing through the last vertex of the lens. This sphere I shall call the "vertex sphere" (see Fig. 2). The centre of this small circle must be on the chief ray of the oblique pencil. When we measure from this point, the correction to be added on the right of equations (17) and (18) is

$$\frac{h^2}{2} (1/a_1 + D + R)^2 (\delta - \rho_3) \text{ or } \frac{h^2}{2} Q.$$

The convergences measured from the vertex sphere are therefore

$$\text{Primary} \qquad \frac{1}{a_3'} = \frac{1}{a_1} + D + R + h^2 (P + Q/2).$$

$$\text{Secondary} \qquad \frac{1}{a_3'} = \frac{1}{a_1} + D + R + h^2 (S + Q/2).$$

Taking the arithmetic mean as the convergence of the circle of least confusion, we have

$$\frac{1}{a_3'} = \frac{1}{a_1} + D + R + \frac{h^2}{2}(P + S + Q) \qquad \text{.....(19).}$$

If this is to be independent of h, we must have

$$P + S + Q = 0 \qquad \text{.....(20).}$$

Equation (20) expresses the condition that must be satisfied if the circles of least confusion of all oblique pencils of small aperture are to lie on the conjugate image sphere and so to focus on the macula. Putting in (20) the values of P, S, Q given above, the final equation is

$$\begin{aligned}
&\left(\frac{\delta - \rho_3 - R}{\mu_1} - \frac{D}{\mu_1 - 1}\right)^2 \{(\rho_3 - 1/a_1)(\mu_1^2 - 1) + (\mu_1 + 1) D\} \\
&\quad + \left(\frac{\delta - \rho_3}{\mu_2} - \frac{R}{\mu_2 - \mu_1}\right)^2 \left\{(\rho_3 - D - 1/a_1)\left(\frac{\mu_2^2 - \mu_1^2}{\mu_1^2}\right) + \left(\frac{\mu_2 + \mu_1}{\mu_1}\right) R\right\} \\
&\quad + (\delta - \rho_3)^2 \left(\frac{\mu_2^2 - 1}{\mu_2^2}\right)(-\rho_3 + D + R + 1/a_1) \\
&\quad + (\delta - \rho_3)(1/a_1 + D + R)^2 = 0 \qquad \text{.....(21).}
\end{aligned}$$

On simplifying, this becomes

$$\begin{aligned}
&\rho_3^2 \left[\frac{3(\mu_1 + 1) D}{\mu_1} + \frac{3(\mu_1 \mu_2 + 1) R}{\mu_1 \mu_2}\right] \\
&+ \rho_3 \left[\frac{2(2\mu_1 + 1) D^2}{\mu_1(\mu_1 - 1)} + D\left\{\frac{2(2\mu_2 - 1) R}{\mu_1 \mu_2} - \frac{4(\mu_1 + 1)\delta}{\mu_1} - \frac{2(2\mu_1 + 1)}{\mu_1 a_1}\right\}\right. \\
&\qquad \left. + \frac{2(2\mu_2 + \mu_1) R^2}{\mu_1 \mu_2 (\mu_2 - \mu_1)} - \frac{4(\mu_1 \mu_2 + 1) R\delta}{\mu_1 \mu_2} - \frac{2(2\mu_1 \mu_2 + 1) R}{\mu_1 \mu_2 a_1} - \frac{1}{a_1^2}\right] \\
&+ \frac{(\mu_1 + 1) D^3}{(\mu_1 - 1)^2} + D^2 \left\{\frac{(\mu_1^2 - 3\mu_1 - 2)\delta}{\mu_1(\mu_1 - 1)} + \frac{2(\mu_1 + 1) R}{\mu_1(\mu_1 - 1)} - \frac{\mu_1 + 1}{(\mu_1 - 1) a_1}\right\} \\
&+ D\left\{\frac{(\mu_2 - \mu_1 - 2) R^2}{\mu_1(\mu_2 - \mu_1)} + \frac{2(\mu_1 \mu_2 - \mu_2 + 1) R\delta}{\mu_1 \mu_2} + \frac{(\mu_1 + 1)\delta^2}{\mu_1}\right. \\
&\qquad \left. - \frac{2(\mu_1 + 1) R}{\mu_1 a_1} + \frac{(4\mu_1 + 2)\delta}{\mu_1 a_1}\right\} \\
&+ \frac{(\mu_2 + \mu_1) R^3}{\mu_1(\mu_2 - \mu_1)^2} - R^2 \left\{\frac{\mu_1(\mu_2 - \mu_1) + 2}{\mu_1(\mu_2 - \mu_1) a_1} + \delta\left(\frac{2(\mu_2 + \mu_1)}{\mu_1 \mu_2 (\mu_2 - \mu_1)} - 1\right)\right\} \\
&+ R\left\{\frac{2(2\mu_1 \mu_2 + 1)\delta}{\mu_1 \mu_2 a_1} + \frac{(\mu_1 \mu_2 + 1)\delta^2}{\mu_1 \mu_2}\right\} + \frac{\delta}{a_1^2} = 0 \qquad \text{.....(22).}
\end{aligned}$$

We may now put in numerical values of the indices of refraction and of the distances of the centre of rotation and the object from the lens.

I assume, as usual, that the distance d of the centre of rotation from the last vertex of the lens is 25 mm., so that $\delta = 1/d = 40$ dioptres. I take the reading or working distance to be 3 dioptres, so that $1/a_1 = -3$. The distance a_1 is measured along an incident ray from the lens to an object point.

In the British Patent Specification No. 225,425, by the United Kingdom Optical Co. Ltd. and Alfred Henry Emerson, the following particulars are given of two glasses by the use of which "a maximum change of reading addition may be made with a minimum of chromatic aberration":

Boro-silicate crown, $n_d = 1{\cdot}508$, $\nu = 63{\cdot}9$.
Dense Barium crown $n_d = 1{\cdot}6086$, $\nu = 57{\cdot}6$.

I shall take for my refractive indices

$$\mu_1 = 1{\cdot}508 \quad \text{and} \quad \mu_2 = 1{\cdot}6086.$$

Putting these values in equation (22), it becomes

$$\begin{aligned}&\rho_3^2\,[4{\cdot}9894\,D + 4{\cdot}2367\,R]\\ &+ \rho_3\,[10{\cdot}485\,D^2 + D\,(1{\cdot}828\,R - 250{\cdot}12) + 38{\cdot}725\,R^2 - 211{\cdot}49\,R - 9]\\ &+ 9{\cdot}7188\,D^3 + D^2\,(6{\cdot}5479\,R - 207{\cdot}1) + D\,(-12{\cdot}52\,R^2 + 69{\cdot}908\,R + 2021{\cdot}9)\\ &+ 204{\cdot}21\,R^3 - 939{\cdot}17\,R^2 + 1680{\cdot}6\,R + 360 = 0 \qquad \text{.....(23).}\end{aligned}$$

Discussion of the equation

The equation is a quadratic in ρ_3 and from it, D and R being given, we can find the curvature of the third surface. The curvatures of the first and second surfaces can then be found by means of the first order equations (5) and (6). For any given values of D and R we can thus make a bifocal lens which has the property that the circles of least confusion of all oblique pencils of small aperture lie on the conjugate image sphere and so focus on the macula.

I have plotted in Fig. 4 curves showing the relation between ρ_3 and D for five values of R, viz. 0, 1, 2, 3, 4 dioptres.

The power D is plotted as abscissa and ρ_3 as ordinate. The scale divisions for D are ten times as big as those for ρ_3. There are two values of ρ_3 for every possible value of D and consequently two forms of lens. The steeper curved lenses are commonly called the "Wollaston" type and the flatter lenses the "Ostwalt" type.

Each curve has an asymptote parallel to the axis of ρ_3 given by

$$D = -\frac{4{\cdot}2367}{4{\cdot}9894}\,R.$$

When $R = 0$ the curve is very nearly an ellipse; if the working distance were great or the incident light parallel, the terms involving $1/a_1$ would vanish and the equation would be of the second degree and the curve an ellipse.

There is a double point when D is a small + quantity and ρ_3 = nearly 40.

The values of ρ_3 become imaginary when D lies outside limits of about + 10·75 and − 24. When $R = 1$ or 2 the curve is continuous but the double point has disappeared; the + limit has become less and the − limit has increased.

At some point between $R = 2$ and $R = 3$ the curve breaks up into two branches, the one on the right being a closed curve. As R increases, the closed curve shrinks and by the time $R = 4$ it has disappeared leaving only the branch on the left.

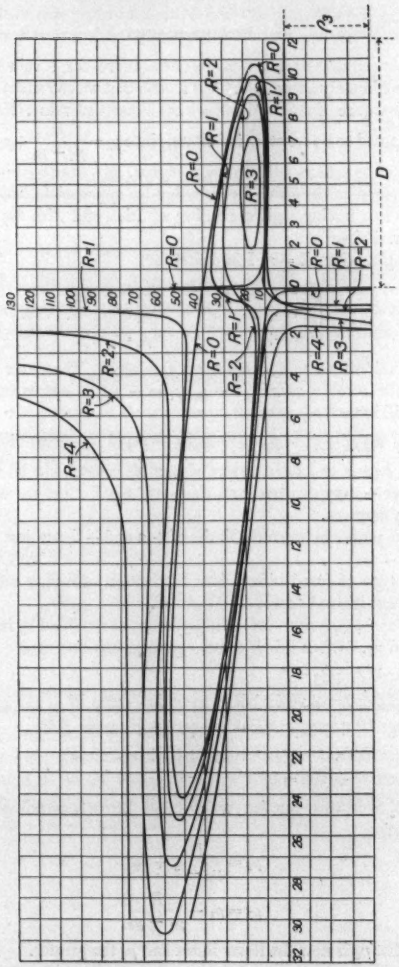


Fig. 4.

BIFOCAL BLANKS

It will be noticed that the lower limbs of the curves for + values of D have parts which are approximately horizontal. This means that we can make a series of lenses in which the curvature of the third surface has a constant value. Now if the reading addition is supposed to be constant it follows, since $\rho_2 = \dfrac{R}{\mu_2 - \mu_1} + \rho_3$, that the curvatures of the second surfaces will also be constant. We can thus make a blank consisting of two surfaces on which by grinding an appropriate first surface we can make a bifocal lens having a given reading addition and with any distance power between certain limits.

As an example, when $R = 2$, I find that for

$$\left.\begin{matrix} D = +2 \\ D = +4 \\ D = +6 \end{matrix}\right\} \text{ we have } \left\{\begin{matrix} \rho_3 = 9{\cdot}17 \\ \rho_3 = 9 \\ \rho_3 = 9{\cdot}07 \end{matrix}\right\}.$$

Taking the middle value $\rho_3 = 9$ we get $\rho_2 = 28{\cdot}88$. The outer or distance part of this blank will have the power $(1 - \mu_1)\rho_3 = -4{\cdot}57$ dioptres and the middle or reading part will have the power

$$(\mu_2 - \mu_1)\rho_2 + (1 - \mu_2)\rho_3 = 2{\cdot}905 - 5{\cdot}476 = -2{\cdot}57 \text{ dioptres.}$$

If the distance lens is to have a power of + 2 we must grind on the blank a first surface of power + 6·57 dioptres, and this surface will make the reading lens have a power of + 4 dioptres.

Any distance power between the limits of + 2 and + 6 can thus be made on this blank.

In another type of fused bifocal lens the button of higher refractive index is fused into the first instead of the third surface.

To suit this type, equations (22) and (23) must be modified by interchanging the values of μ_1 and μ_2. When this is done, equation (23) becomes

$$\begin{aligned} &\rho_3^2\,[4{\cdot}865\,D + 4{\cdot}237\,R] \\ &\quad + \rho_3\,[8{\cdot}615\,D^2 + 1{\cdot}662\,DR - 243{\cdot}7\,D - 37{\cdot}9\,R^2 - 211{\cdot}49\,R - 9] \\ &\quad + 7{\cdot}043\,D^3 + 5{\cdot}329\,D^2R - 160{\cdot}31\,D^2 + 12{\cdot}981\,DR^2 \\ &\quad + 72{\cdot}98\,DR + 1965{\cdot}4\,D + 191{\cdot}4\,R^2 + 1027{\cdot}6\,R^2 \\ &\quad + 1680{\cdot}6\,R + 360 = 0 \end{aligned} \qquad \text{.....(23').}$$

Having found ρ_3 from this equation, we can find the corresponding values of ρ_1 and ρ_2 from the relations

$$\rho_1 = \frac{D}{\mu_2 - 1} + \rho_3 \qquad \text{.....(5'),}$$

$$\rho_2 = \rho_1 - \frac{R}{\mu_1 - \mu_2} \qquad \text{.....(6'),}$$

not forgetting that μ_2 is now the lesser index and μ_1 the greater.

As an example of this type of bifocal, I find that the values of ρ_3 obtained from equation (23′) for $R = 2$ and $D = 2$, 3 and 4 and the corresponding values of ρ_1 and ρ_2 are

D	R	ρ_3	ρ_2	ρ_1
2	2	19·49	3·547	23·43
3	2	18·21	4·235	24·12
4	2	17·52	5·510	25·39

The values of ρ_1 and ρ_2 are not sufficiently constant to make a good blank, but taking the values for $D_3 = 3$ the powers of the first two surfaces forming a blank would be

Distance $(\mu_2 - 1)\rho_1 = 12{\cdot}25.$

Reading $(\mu_1 - 1)\rho_1 + (\mu_2 - \mu_1)\rho_2 = 14{\cdot}68 \quad 0{\cdot}425 = 14{\cdot}25.$

These two surfaces give a reading addition of 2, and for distance powers of + 2, + 3 and + 4 the third surfaces must be − 10·25, − 9·25 and − 8·25 dioptres respectively.

Correction for plane object

If the object is supposed to be a plane at right angles to the optical axis at a distance $-a_0$ from the first vertex measured along the axis, then $1/a_1$ in equation (19) must be replaced by $\frac{1}{a_0} - \frac{h^2}{2}T$ where

$$T = \frac{D}{a_0^2(\mu_1 - 1)} + \frac{\rho_3}{a_0^2} + \frac{\{\delta - (D + R)\}^2}{a_0},$$

and the equation (19) will then become

$$\frac{1}{a_3'} = \frac{1}{a_0} + D + R + \frac{h^2}{2}(P + S + Q - T) \qquad \text{.....(19′)}$$

Prismatic effect

The investigation shows that to obtain the best images of objects off the axis it is essential that the three surfaces of the reading lens must be co-axial and the axis must pass through the centre of rotation of the eye.

When the lens is decentered it is impossible to make the circles of least confusion of all oblique pencils lie on the conjugate image sphere, and consequently prismatic effect produced by decentration of the lens as a whole or by decentering one of the surfaces must always be accompanied by an inferior image.

DISCUSSION

Mr T. Smith: I am interested to see that Mr Whitwell is extending his work on spectacles to the forms using more than one kind of glass. From both the algebra and the arithmetic of the paper it appears to me that the author has given himself an unnecessary amount of work—the theory of these lenses is essentially the same as that of thin lenses such as telescope objectives.

There are two directions in which it would be interesting to extend the work. The claims of the least circle of confusion to be taken as the fundamental quantity in the design might well be examined from the point of view of path difference, and the problem of astigmatic and decentered lenses is also worth investigating. Neither should offer any real difficulty.

As far as I can see from the paper, the author has assumed, not proved, that the lens surfaces must be coaxial, and that this common axis must pass through the centre of rotation of the eye. The conclusion is doubtless correct as long as such a lens is suited to the wearer's needs.

I am sure the paper will be widely studied and appreciated, and I should like to express my personal interest in Mr Whitwell's work.

Mr Whitwell: In reply to Mr Smith I agree that "the theory of these lenses is essentially the same as that of thin lenses such as telescope objectives" provided we make the chief rays of the imaging pencils pass through the point where the centre of rotation of the eye would be.

When the three lens surfaces are not co-axial or when the lens is decentered, I found that the aberration terms (see equation (19)) contained the first power of the aperture as well as the second and that the circles of least confusion of all oblique pencils of small aperture could not be made to lie on the conjugate image sphere unless the three centres of curvature and the centre of rotation were in one straight line.

I am at present studying the problem of spectacle lens design from the point of view of path difference as suggested by Mr Smith and beg to thank him for his expression of interest in my work.

NEW KNOWLEDGE OF OLD TELESCOPES

(A SUPPLEMENT TO THE THIRD PAPER ON THE COURT COLLECTION)

By THOMAS H. COURT and MORITZ VON ROHR

MS. received, 30th October, 1930. Read, 11th December, 1930.

ABSTRACT. Three documents referring to the development of the telescope are discussed. They are arranged with respect to the gradual development of this optical device and its production in London. Document I is a letter written by Beal on 24th November, 1656. It refers only to the Dutch or Galilean telescope and gives interesting details regarding the hand-glass used by Gustavus Adolphus and the taste shown by the English opticians of 1656 in decorating the bodies of Galilean telescopes. Document II is a photographic reproduction of J. Wiesel's price list of 1647. It is by far the most important from the historical standpoint. From it we learn some details of the industrial development of Wiesel's telescope factory in Augsburg at the surprisingly early date of 1647. Document III is a letter written by the well-known London optician Richard Reeves on 11th March, 1660. It informs us of the state of telescope manufacture in London at that time. The prices given for the three kinds of telescopes—Dutch or Galilean, Keplerian, and Schyrlean—are quite reasonable and much lower than those quoted in Document II.

Of the three important documents* forming the basis of this supplementary paper, we shall first deal with the English letter of 24th November, 1656, because it refers to earlier instruments as well as to an earlier state of knowledge.

Document I. Letter written by Beal on 24th November, 1656

Nov. 24. 1656.

The { Horseman's / Seaman's / Galilaean } Perspective.

Sr By the carrier I send you two trifles, the one beautiful, the other most ugly. The beautiful is a phantastical present for an effeminate person & unfit to be sent back to London from whence it came. The ugly present is a philosophical design. By the first I do compliment with you and beg your pardon for it. The other, if I mistake not, is a fit present for a warlike prince such as our Protector, to whom I wish you would present it reformed & amplified as I shall hereby describe, if you will afford patience to hear it. My brother Capt. Rd Beal, who had the command of the Royal frigate Charles about 30 years ago, was afterwards [groom] of the bedchamber to Marq. Hamilton, & commissary to those unhappy English forces that were sent to King Gustavus Adolphus of Sweden. He

* As I am far from derogating from my friend's merits, I feel obliged expressly to state that all the three documents of the years 1647, 1656, 1660 were discovered by Mr T. H. Court among the manuscripts of the British Museum. I am sure every reader must admire his devoted perseverance and congratulate him on the splendid success of his historical work on the development of optics. M. v. R.

observed that this valiant King was short-sighted almost purblind. For help he used a short perspective so concealed in his hand that it was not discerned: whilst he seemed to regulate the brim of his beaver, he used this perspective. This which I send is just fit for the view of an army or a ship at sea.

On horse back or in the speed of a coach or in a ship you may see by this perspective clearly. My brother was a most ingenious man, as by other proofs I mean hereafter to acquaint you. This he added to perfect the more imperfect perspective of that King. And in this ugly wood he shaped his phantasy with his own hands. The hollow glass I broke in my pocket. Now my proposal is this: If you get a glass perfectly hollowed on both sides, this broken glass will direct (and by the depth of the concavity & art, you may increase or diminish at great diversity). By the imperfect concave which I send also, you will see what to decline. So the burning glass at the broad end may be made larger or more gibbous for variety. To fit it for variety the concave glass being excellent should be put into an ivory box (rather than silver or gold) of less than an inch length, to be covered on the outside with an ivory screw for defence (it being so thin as may easily be broken). And at the other end of that ivory box to be screwed into several such *handles* as I send you but made beautiful as of paste board and gilt leather; or of rich velvet, or so wrought as the other trifle is. This a prince will not disdaign. The *handles* as I have last called them should be of three sorts at least.

First no longer [II] than this which I send you fit for a horseman to view an army: or in recreations to view a whole herd of deer at once. A lady may find it a servant for Hyde Park shows or other pleasures.

The second *handle or shaft* may be of burning glass and of more length fitted to view a navy at sea.

The third *shaft* should be a most perfect perspective which might be, as a staff, made taperwise, as the walking staves were four years ago. The footing of the staff being screwed off and this ivory joint screwed on at the smaller and lower end. And a perfuming knob screwed off from the top of the staff a glass there remaining would make the body of the staff a complete perspective. To perfect this also a glass out of a spectacles fitting the eye sight [?] of the person to whom you give it put into the remote end of the perspective, that is the over end of the staff would perfect the instrument to a kind of a selenoscope by which a prince would take pleasure to behold the terrestrial and (for ought we know) inhabitable part of the moon. To do this you should first get variety of glasses in perfection. Another Artist [?] not acquainted with your purposes: should fit the principal screw into moveable shanks of the usual paste board. This will cost little. Then when you have found the just length and use you should speedily make up your design. And take heed that your purposes be not discerned nor the business too hastily made common. For it will be of some use to my purpose of dispatch of intelligence in the offices of communication and accommodation, as you shall be more fully informed shortly G[od] W[illing]. Before you can prepare these three perspectives which I will call a Horseman's perspective, a Seaman's perspective and a little Galilaean perspective, my design intended for his Highness may surprise you. I note that a perspective is helped if the inside of the shaft be of the blackest hue. And that some taper canes of the longest joints are fittest for this use. The outside of canes being strongest or box or for better colour tough yew or plum tree made strong like a trunchion [III] fitted for a proper man's hand. It should be three or four feet long [0·9 to 1·2 m.].

You may add a fourth device. To a tall staff of Authority containing 6 feet length [1·83 m.] and further yet to a soldier's longest pike. The first ivory screw serving to all. And to this a magnifying glass screwed on close to the concave for a contrary use. I send you my magnifying glass that you may see how short and compendious the first screw may be for all these uses. For that I write of putting a spectacle glass on the remote end of the walking staff perspective I leave to your choice. But of mine own head I did put one of

Sir H. Wotton's spectacles to a perspective of two feet length for his use. He found such an incredible perfection in the glass, that he was almost ravished at the conceit, as if by a miracle he had been restored to his youngest sight. To us that were young it had the same advantage: I brought the perspective with me to King's College Cambridge and there we could discern a bird's eye and all other motions of the bird, upon remote spires, when no saleable perspective did discover the spire itself. Other spectacles have failed me of that perfection so that some advantage was peculiar to the glass being perfectly gibbous in a most equable and meet dimension.

This magnifying glass is so gibbous that it is strong enough to be screwed at the end of the concave glass for a defence to it in the pocket. This adorns, defends and also multiplies variety of uses, but then it should be fastened into the first screw with a very strong wire.—And the distance between the magnifying glass and the concave glass (when both are screwed together) of just proportion for use as you see here between the magnifying gibbosity and the black foil [?]. Also there is a manifold use of the concave at this distance placed to look through the narrow gibbosity. And also as much use of looking through the gibbosity and also through the first screwed concave glass to other [IV] objects at fit distances placed. When you have fitted the screws in ivory by the help of several movable pasteboard stems with glasses at the end of some of them, you may in one quarter of an hour find many uses and fit the length but take heed of hurting the eye by trying too much at one time. So much of this matter.

P.S. Since I sealed up the other it came in my mind to entreat you to put this concave glass or a better into an ivory screw; and that screw to be screwed into a shank of plain leather containing the burning-glass at four inches and a half or 5 inches [11½ to 13 cm.] distance from the concave glass. If the shank be movable at the distance of half an inch then it is better. Also to be screwed in a longer perspective shank of movable joints in which the annexed spectacle glass is fastend as at the end. This is like an additional explication of the directions contained in the letter and will give some light unto you to discern what may be corrected or amplified in the greater design. You will pardon this boldness and I will study serious thankful [?] Your servant.

The first end should be so wide to contain the whole eye fitly and fully as in the ugly wood. And the burning glass should be left at largest not covered in the borders but by a strong wire.

This letter, written by the brother of a Capt. Beal, is not signed, neither is his correspondent known to us. The writer seems to have been a younger brother of the Captain, and as he had access to King's College, Cambridge, he may possibly have been there as a student. His correspondent evidently held some position bringing him near the Lord Protector, for in the letter he is asked to present to this ruler one of the new telescopes when properly finished.

R. Beal, as commissary to the English force* which was sent to aid Gustavus Adolphus at the beginning of his German campaign in the Thirty Years' War, had been able—apparently in the years 1631 and 1632—to make some observations of

* As soon as it was known that the first Duke of Hamilton was the commander of this expedition, details were easily obtained from the *Enc. Brit.* 11th ed. **12**, 886 *a*. It appears that the force was sent over in 1631 and consisted of 6000 men; but as they had to guard the Magdeburg country which had been completely ruined (Magdeburg having been invested and stormed in 1631), these men suffered dreadfully from starvation and disease, and the expedition became a total failure. In September, 1634, the Duke of Hamilton returned to England. In the civil wars he was beaten by Cromwell at Preston (17th to 19th August, 1648), taken prisoner, and found guilty of treason. He was executed on 9th March, 1649.

the habits of this warlike king. He corroborates the well-known fact that this king was short-sighted and relates that he used to employ a very small Dutch telescope which could be concealed within his hand, in order that he might unostentatiously overcome his defect. This glass must have been of small magnification, as he used a similar one on horseback, in a coach at full speed, and on board ship.

This letter enlarges our knowledge of early exceptionally short Dutch telescopes. Taking 1631 to be its date, this instrument comes between the short form of Chorez in 1625 and the small telescope of A. Kircher in 1643. It must have been very short in comparison with the instruments then in general use.

It seems as if Beal took objection to the small magnification, and at the same time designed a useful rest for the eye when using the instrument.

Acting on his brother's ideas, the writer, Beal, planned different telescopes—all of them after the Dutch plan—which had in one case a rigid body well blackened inside. Probably the inventor was dissatisfied with the usual telescoping cardboard tubes which were then regularly made in England (after the Venetian fashion in use before 1618), considering them to lack rigidity.

The four different forms had different magnifications and we obtain the following scanty details from Beal's letter.

1. *The Horseman's Perspective.* No length given (perhaps 1 foot). Destined for a reconnoitering officer on horseback, for deer stalking, or for racing, it had a low magnification.

2. *The Seaman's Perspective.* No length given (perhaps 2 feet, as it was to be longer than the other; see also Sir H. Wotton's specimen).

3*a*. *The* (shorter) *Galilaean Perspective.* Three or 4 feet long with slightly tapering shaft and arranged to be applied to a walking-stick; this had to be unscrewed when serving as a telescope. The writer of the letter intends employing it also for some secret purpose and to offer it "in the offices of communication and accommodation" for his "purpose of dispatch of intelligence." One might be induced to think of a kind of optical telegraph, but that is only a supposition.

3*b*. *The* (longer) *Galilaean Perspective.* Six feet or even longer. We may observe with regard to 3*a* and 3*b* that the distinction "Galilaean" is used here in a way differing somewhat from the ordinary use; it means here telescopes for astronomical observations and clearly shows the appreciation for Galileo's work as an astronomer even so late as the end of 1656.

As the writer recommends the use of one and the same concave lens with all the three tubes, the magnification will have varied nearly in proportion to the length of the rigid tubes.

The writer relates Sir H. Wotton's high appreciation of the value of such telescopes. If this is the same Sir H. Wotton as is referred to in General Waterhouse's paper* of 1902 before the Royal Photographic Society, we must attach some merit to the instrument in question. The experiments made in Cambridge are not convincing, as there is no statement that the new and the commercial perspectives mentioned were of the same or nearly the same magnification.

* *Phot. Journ.* **42** = (2) **26** 1902 (4–21); 10 figs. See pp. 19, 21.

The aperture of the rigid telescope he put together for Sir H. Wotton with a body of 2 feet in length was that of a spectacle glass of about $1\frac{1}{2}$ inches.

That no particular care was taken in selecting the "burning glass" or objective does not give us a high opinion of the writer as an optical expert, neither does he seem to have known the simple rules for determining the magnification of a telescope of Dutch construction.

Document II. J. Wiesel's price list of 1647

The second document to which we wish to draw attention is the Latin text of Wiesel's price list of 1647; it may have been copied from Sir Charles Cavendish's correspondence referred to in the Third Paper on the Court Collection.

The importance of this document is much greater than that of the former, as it gives us definite knowledge of a stage of the development that till now was deemed irrevocably lost. Therefore we not only reproduce the Latin text*, but also add the following notes. The price list tells us of three different kinds of telescopes.

1. The oldest form contains a positive lens as objective and a negative lens as eyepiece, i.e. the old Dutch construction, then nearly 40 years old.

They are listed in three different groups:

(*a*) Short hand-glasses of 6 to 9 inches in length, apparently with small magnifications and not very highly valued; their price was in round numbers £2 and £2. 10*s*.

(*b*) Telescopes of medium size 3 feet long for about £3, 6 feet long for about £5. 13*s*. 4*d*. to £7.

(*c*) Telescopes of great length 10 feet long for about £14, 12 feet long for about £19, 14 feet long for about £23. 13*s*. 4*d*.

They were evidently made with draw tubes (as made in Venice before 1618); and we may conclude that their magnification varied with their length, for the 6-foot telescope is said to command distances up to 18 to 23 miles, and that is only half the distance given for the next group, viz. 37 to 46 miles. Such distances seem rather exaggerated, as the air will seldom allow objects so far away to be distinguished clearly.

Wiesel was quite aware of the fact that this construction was the oldest of the three.

2. A more recent form of telescope is made with several (i.e. more than two) glasses and is chiefly intended for observing the stars. Evidently this is Kepler's form of 1611; its advantages were also known to Gascoigne in 1642. We are at present unable to state exactly the date when these instruments were first offered commercially. From the Latin expression used by Wiesel we may conclude that such glasses had been introduced some time—perhaps 10 years—before.

He is well acquainted with the principal advantage of this construction, the much larger field of view. He states that his specimens had thirty times as large a

* 1 sovereign = 2·11 Ducati; 4·6 stat. miles = 1 Milliarium Germanicum; 1·03 English foot = 1 pes (may be considered equal for short lengths); 3 inches = 1 Spithama.

Ex Epistolâ Hamburgo missâ. Sept. 25. 1647.

Apographum

Tabulæ transmissæ a Johanne Wiselio, Optico Augustano pretium Tuborum opticorum referentis.

De prebio horum Tuborum quæritur: Ubi notandum tria eorum genera esse.

Unum Antiquiorum ex duobus vitris constantium, convexo altero altero concavo, quorum pretium pro longibudine ipsorum attollitur: quidam enim tres pedes longi 6. Ducatis constant: quidam sex pedes, pretio inter 12. et 15. Ducatos intermedio. Hisce coelo sereno ad 4 aut 5. milliaria Germanica visus provogari potest. Alij 10. 12. 14. pedes longi, qui visum in 8. 10. milliaria Germanica visus provogari potest. Alij 10. 12. 14. pedes longi, qui visum in 8. 10. milliaria proferunt, et 30. 40. 50. Ducatis veneunt. Sed et habentur viliores, duarum vel trium spithamarum longitudine, pro 4. aut 5. Ducatis. Et hoc primum genus est Antiquius.

Photographic reproduction of Wiesel's price list

Horum est Recentionum, ac primo quidem ex diversis vitris Compositorum, quibus objecta conspiciuntur inversa, et utplurimum ad Aedificiorum contemplationem adhibentur: Nobilomnibus eximium in terra quoq; usum praebens interdiu, quoniam per illos spatium amplissimum plurimaq; res in illo contenta unico intuitu offeruntur: cum è contra vespertinis illis vix propinqua horum pars appareat. Eorum longitudo 10. aut 14. pedes non excedit quorum minimum pretium ad 50. aut 60. ducatos assurgit.

Secundum genus sequitur: Hi recentissimi sunt, ex diversis vitris compositi: quibus totum exercitum 7. aut 8. circiter milliaria simul perspicuè contueri et quam clarissimè lustrare licet: Et quidem non ut in priori situ inverso sed recto et genuino. Per hos in plenilunio mirabiles figuras et circa Jovem non tantum 4. sed vel novem Stellas minores conspiciuntur. Hujusce tubi protracti et ad usum accommodati 10. 12. vel 14 pedum longitudo aut pretium non infra 120. Ducatos. Neq; ullus adhuc ex hoc genere haetenus in lucem prodiit aut venalibus est, nuperrimè quippe inventus.

(retouched in order to ensure better legibility).

field as those in group 1. By a simple computation of the field of view of glasses in group 1 with medium and strong magnifications (and comparatively modest apertures of the objective) we find that the observer's pupil determines the angle of view. For terrestrial use during the bright hours of the day, when the pupil is small, the field will therefore be more restricted than in celestial observations during the night. For the Kepler telescopes in group 2 the angle of view depends on the diameter of the field lens in the eyepiece, and it is possible to account for the ratio 1 : 30 given by Wiesel. He expressly mentions that, as the field was larger, these glasses were formerly also used for terrestrial observation, the observer being willing to put up with the inverted position of the image.

Of these telescopes of great length he listed only one 10 feet long for about £23. 13*s*. 4*d*. and one 14 feet long for about £28. 10*s*.

3. The third type listed were quite new. Formed of several lenses, they present the image in an upright position and show also a large field of view. These are evidently A. M. v. Schyrle's terrestrial telescopes, described two years previously in his *Oculus Oenoch & Eliae*. This is the first time that we know that this first form of the terrestrial telescope with several positive lenses was commercially offered. This construction was to reign with its improvements undisturbed for 250 years, until it was gradually superseded by Abbe's form of binocular field-glasses with erecting Porro prisms. Wiesel pointed out the advantage gained for military purposes by the increase of the field of view, and, as we know, his expectation was to be amply justified. He also mentioned astronomical uses for this construction, but this was early found to be a mistake, for the astronomers preferred the construction of type 2.

The prices given for glasses in group 3 are as follows: 10 feet, 12 feet and 14 feet long, from £57.

It is possible that Sir Charles Cavendish actually bought such an instrument of group 3, and that this served as a model for Cocks's first terrestrial telescopes made according to v. Schyrle's construction.

In this price list Wiesel does not give any details of the construction of his instruments; he naturally had no intention of helping competing opticians.

It is of interest to know that towards the end of the Thirty Years' War the optician Wiesel in the Free Town of Augsburg was able to offer these different kinds of telescopes at such high prices; and we can understand how 13 years after the issue of this price list Wiesel's portrait (engraved in 1660) was willingly bought by his admiring countrymen.

Document III. Letter written by Richard Reeves on 11th March, 1660

We are happily in a position to throw some light on the introduction of these telescopes into England, as one of us has found a copy of a letter written by R. Reeves himself on 11th March, 1660.

The text (transcribed into modern English) is the following:

Mr. Richard REEVES (our famous glass
-grinder) his letter to me [Dr. POWER, F.R.S.]
about my Telescope 11th March (1660).

Sir,

According to your brother's request who was lately with me I shall give you an account of telescopes I have which I conceive are fittest to your intent for the observation of the eclipse* of the sun by Mercury. The longest glasses are the best in regard they greaten the object most and so discover the minute parts of it more particularly. But Galilaeus that he observed with were made with a concave glass next the eye and a convex glass next the object which because if the tube be of much length they take in but a little compass, and are not therefore so useful in observing the heavenly bodies as those more lately invented which are made with 3 or 4 convexes and never a concave because these sorts of telescopes (as is demonstrable by reason and confirmed by experience) will take in 40 times more of an object than the other. I have here indorsed sent you for your satisfaction all the prices of my glasses both microscopes and telescopes and shall be glad to serve you with any you please to write for, assuring you they shall be as good and as well wrought as any England can afford. Sir I presume there will be some long observations of this strange eclipse made at my house several men of Art in town determining to meet there that day to the same purpose, what observations they make I shall (if you desire it) send you an account of, and shall in lieu thereof beg the like favour of you. Sir no more at present but that I desire to be

your servant to my power.

Richard REEVE

11th March (1660)

Telescopes with Concaves

		£	s.	d.
In Length	2 Inches	0	10	0
	3 ,, ...	0	10	0
	6 ,, ...	0	10	0
	1 Foote	0	10	0
	2 ,, ...	0	10	0
	3 ,, ...	1	0	0
	4 ,, ...	2	0	0

Telescopes with Convexes inverted and erected

	£	s.	d.
2 Foote	4	0	0
3 ,, ...	5	0	0
4 ,, ...	6	0	0
5 ,, ...	8	0	0
6 ,, ...	10	0	0
9 ,, ...	12	0	0
12 ,, ...	15	0	0
24 ,, ...	20	0	0
36 ,, ...	30	0	0

Microscopes of several sizes

£	s.	d.
3	0	0
4	0	0
5	0	0
6	0	0

* Probably in May, 1661. M. v. R.

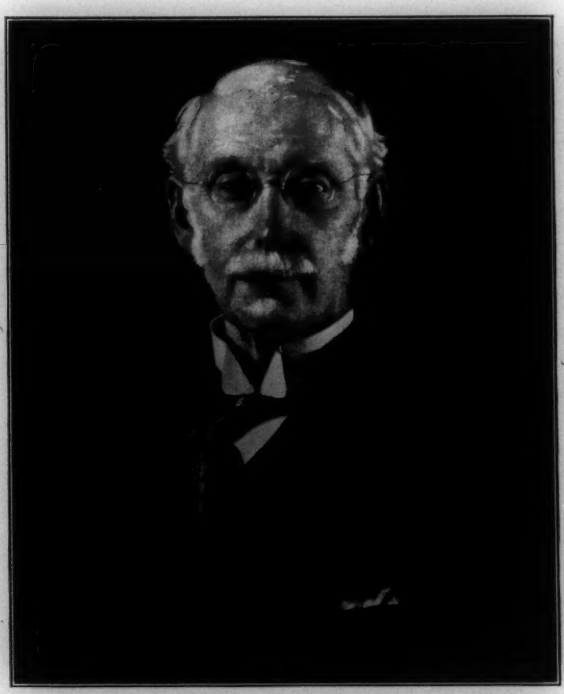
We may see that Reeves' description is in good accordance with Wiesel's catalogue of 1647. The glasses with three convexes are apparently of Kepler's form, the instruments with four convexes are telescopes made according to v. Schyrle's construction. He points out their advantage in yielding a larger field, as the Augsburg optician had done; he even gives the extent of the field of view in the later glasses as 40 : 1 compared with the astronomical telescopes of Dutch construction, as employed by Galileo in his discoveries.

As to the first list, we may remark that such an early offer of very short Dutch glasses (2 and 3 inches) is very remarkable. As these prices are very low, the glasses no doubt had a plain exterior. (From Beal's letter we know that in 1656 beautiful mountings were made by London artists in paste-board and gilt leather, in rich velvet, and in other designs.)

Reeves' prices are very moderate. That an optician of his standing still offered Dutch glasses of 4 feet length shows the strength of the old custom.

As to the second list, containing Kepler's and v. Schyrle's glasses, we note that there is no difference in the price. We may suppose that the Schyrlean form was used for the shorter (travelling) telescopes, whereas the longer, and especially the very long, tubes were stationary and when intended for astronomical observations were no doubt made after Kepler's plan. It is to be noted that the prices are much more reasonable than those of Wiesel.

BLANK
PAGE



THE HON. SIR CHARLES A. PARSONS, O.M., K.C.B., F.R.S.

(By courtesy of *Engineering*.)

To face p. 123]

OBITUARY NOTICE

The Hon. Sir CHARLES A. PARSONS, O.M., K.C.B., F.R.S.

The Optical Industry of this country has suffered a great loss by the death of Sir Charles A. Parsons, which took place after a very short illness on Wednesday, 11th February, 1931, while he was on a cruise to the West Indies.

Sir Charles was the fourth son of the third Earl of Rosse, and was educated privately at Birr Castle in Ireland, one of his tutors being Sir Robert Ball (later the Astronomer-Royal of Ireland). Much of his time not occupied by studies was spent in his father's workshops, where telescope work of one kind or another was always going on. The Earl of Rosse will be remembered as the famous astronomer who made the first 6 ft. reflector with his own hands. The inherited interest in and leaning towards optical matters, always ready to assert itself, led Sir Charles late in life to take up the burden of controlling three optical concerns, in addition to the searchlight mirror department connected with his Turbine Works at Heaton near Newcastle-on-Tyne.

The development of the mirror department which he always regarded as a favoured child of his large industrial family enabled him to give scope to his optical tendencies. He invented a new process of making searchlight mirrors up to 6 and 7 ft. in diameter true to predetermined shape.

With his determination always to get the best and most efficient, he soon recognised the necessity of investigating the glass from which the mirrors were made, with a view to securing greater power in the mirrors of withstanding sudden and great changes of temperature. This investigation resulted in the introduction of a specially heat-resisting boro-silicate glass for these mirrors.

After the mirror department at Heaton had become a thoroughly efficient establishment, he began to look around for a larger field of interest in things optical and for opportunities to help in the development of the optical industry in this country.

In January, 1921, he acquired a large interest in the Optical Firm of Ross Ltd., and became Chairman of the Company. Thus for ten years it was my privilege to work with him. With the close personal touch of business one became more and more fully aware of the greatness of the man, of his charming manner and ever ready help and sympathy, with at the same time a firm demand for a high standard of efficiency. Just as he was always working for economy and efficiency in turbines, so was he always striving for perfection in Ross' instruments concurrent with lowest production costs. His great engineering genius led him to look at glass grinding from an entirely different point of view from that usually adopted by the man in the trade, and the result was a great many valuable suggestions leading to changes in methods and to improved ways of doing things.

He was uncanny in his ability to lay his finger on the weak spot of any contrivance which had been much thought of by its originator, and here his great charm of manner was much in evidence, so that no man hesitated to lay his ideas before him, and all were always the richer for the advice and help unstintingly given. Sir Charles had ready almost at once not one solution only, but several, leading to embarrassment as to which of them to choose.

He took great interest in telescopes, prism binoculars and all products of the firm of Ross Ltd., and was always pressing forward experiments and research. Being an expert experimenter and research worker himself, one of his favourite phrases in discussing any construction or process was—have you tried it?

As soon as Sir Charles became fully acquainted with the work at Ross and with optical glass, he very quickly became interested in the production of optical glass.

The continuance of the Derby Crown Glass Works, which had been founded during the War by Wood Brothers of Barnsley, became at one time very problematical, and Sir Charles, having realised the importance of optical glass in relation to the production of optical instruments, considered it would be a great pity if an organisation so laboriously built up were allowed to disappear, so he saved it by taking this additional burden on himself.

There can be little doubt, however, that he was also greatly attracted to the idea which was always at the back of his mind, namely the making of very large telescope object glasses, and he clearly saw that a necessary condition of success would be the control of the manufacture of the essential optical glass discs.

The Derby Crown Glass Company was taken over by him in July, 1921, and subsequently became the Parsons Optical Glass Company. As might have been expected from a man of his originality of thought and remarkable experimental ability, he soon evolved a method of producing large discs which enabled him to accept an order from Sir Howard Grubb and Sons for the crown and flint discs for the 27 in. Johannesburg Objective, and he succeeded in delivering these discs in the remarkably short time stipulated, namely nine months.

Encouraged by this success he further developed his own original methods so that very much larger discs could be produced. Much experiment and research was needed to arrive at this result; all stages of the manufacture had to come under review. He began with improvements in furnaces to check pot breakages; methods of stirring to obtain greater homogeneity in such large meltings had to be specially designed and many difficulties had to be overcome in the handling of the completed melting. The control of the process of annealing proved to be of no small difficulty. Sir Charles' great gift of solving difficult problems did not forsake him and today glass discs of any magnitude up to 42 in. in diameter are in general production.

Throughout all this work the idea of helping to develop the production of large astronomical telescopes was ever present. The interest which he had found in the various activities of Ross, including as they did the manufacture of lenses of all descriptions, coupled with the certainty of now being able to produce suitable glass discs, stimulated him to look around for an opportunity of taking up the manufacture of large telescopes.

An old-established firm of deservedly high reputation, Sir Howard Grubb and Sons, was taken over by Sir Charles in February, 1925, and became Sir Howard Grubb, Parsons and Co., Newcastle-on-Tyne. In the short period of five years a number of important large telescopes have been made and delivered, including the 36 in. reflector erected at Edinburgh, the largest modern instrument in Great Britain.

It adds poignancy to the deep regret felt for his death that Sir Charles did not live to see the completion of the 74 in. reflector for Toronto now being made at his works. This will be the largest reflector ever made in Europe.

J. HASSELKUS

REVIEWS

An Introduction to Applied Optics—Volume I—General and Physiological. By L. C. MARTIN, D.Sc., A.R.C.S., D.I.C. Pp. ix + 324; 198 Figs. (London: Sir Isaac Pitman & Sons, Ltd., 1930.) Price 21*s.* net.

The author has aimed at producing a text-book on optics which will be of really practical service to those who are interested in optical instruments either as users or makers. He has succeeded in realising this aim in no small measure. The book differs considerably from the more familiar text-books on optics with respect to the arrangement and treatment of the contents, and, unlike many books entitled "Applied Optics," does not deal mainly with the computation of optical systems.

The general plan of the present volume is as follows:

The first chapter deals with the elementary theory of reflection and refraction at plane and spherical surfaces, leading up to the thin lens. Included in the chapter are the fundamental but simple formulae for trigonometrically tracing rays through spherical surfaces. The general theory of optical systems is discussed in Chap. II. This is followed by a well-written chapter on the Physical Study of Light, dealing with wave motion, the nature of light, diffraction, emission, and absorption spectra, etc., forming the physical ground work for the succeeding chapter (Chap. IV) on Optical Images and their Defects.

Chap. V, entitled "Physiological Optics," includes articles on the structure and optical properties of the eye, light perception, visual acuity, and colour vision, and is one of the most interesting chapters in the book. Those who wish to use instruments intelligently will find that much useful information has been condensed in this chapter and will gain by studying it. Chap. VI continues the study of physical optics commenced in Chap. III and covers interference, double refraction, and polarisation.

The seventh chapter discusses the properties of optical glass and contains articles on the manufacture and testing of optical glass and optical parts. The articles on manufacturing processes are of necessity brief but are sufficient to give the reader a connected account of the way in which a good lens is produced, and to impress upon him the respect which is due to the optical components he uses, because of the skill and unceasing vigilance which is bestowed upon them at every stage during manufacture. The author has shown wisdom in including this chapter in the first volume and so providing teachers and students with a ready answer to the inevitable question "How is this made?" The volume concludes with a chapter on the theory and properties of spectacle lenses and two short appendices on the sign convention and wave curvature methods.

This book can be recommended to students who wish to obtain a liberal knowledge of the principles of geometrical and physical optics particularly with a view to applying this knowledge to the study of instruments. It is also a very good reference book for any who are specially interested in instruments. Teachers of elementary optics who study this book will be inspired to enhance the interest of their lectures by bringing them into closer contact with everyday problems, and will find material condensed in this book which would otherwise have to be obtained from several sources which may not always be easy of access. The publication of the second volume will be awaited with interest.

The typography is good and the diagrams have been reproduced clearly.

W. B. C.

Données Numériques de Spectroscopie. Extrait de vol. VII, Années 1925–26. Rédigées par L. BRUNINGHAUS, V. HENRI, et F. WOLFERS. Pp. xix + 514. (Paris: Gauthier-Villars et Cie, 1930.) Price: cloth 190 fr., paper 170 fr.

Since the publication of Bohr's theory of spectra, and more particularly within the last decade, there has been a vast increase in the output of spectroscopic research work. This is well exemplified in the tables under review, which comprise an extract from the *Tables Annuelles de Constantes et Données Numériques* dealing with spectroscopic data only. The corresponding extract from vol. V included 350 pages and covered the six years from 1917 to 1922, while the present extract from vol. VII requires upwards of 500 pages to cover two years only—1925 and 1926. No attempt is made in this publication to give a critical selection of values such as is found in Kayser's monumental handbook. The form is, indeed, quite different, an abstract in both French and English being given of each paper in which data appear accompanied by a reprint of the numerical data themselves. The first 300 pages are devoted to emission spectra which are dealt with in three divisions—the description of spectra including standard lines, X-ray spectra, etc., the effect of excitation conditions and the grouping of lines and bands. A further 166 pages are concerned with absorption spectra. This section has been compiled by Prof. Victor Henri and is enriched by an introduction from his pen outlining the nature of these spectra. The remaining section deals with Stark and Zeeman effect, birefringence, and similar phenomena.

In effect, therefore, these tables comprise a classified bibliography with abstracts of papers which have appeared in the years under review dealing with spectroscopic matters. The list of about 570 journals from which data have been collected, coupled with the statement that others not listed have been examined but have yielded no fruit, would seem to assure the comprehensiveness of these tables, and it may be concluded that they are certain to be of value to all spectroscopists.

F. S.

Band Spectra and Molecular Structure. By R. DE L. KRONIG. Pp. x + 163; 16 Figs. (Cambridge: University Press, 1930.) Price 10*s*. 6*d*. net.

This book originated in a series of lectures given by Dr Kronig at Trinity College, Cambridge, in the Easter Term of 1929. It furnishes a summary, by one who has himself contributed largely to its development, of an important branch of physical inquiry which is proving particularly responsive to wave-mechanical methods of approach.

In five chapters, of approximately equal length, the author deals with (1) diatomic molecules—energy levels and their classification by quantum numbers, (2) fine structure and wave-mechanical properties of the levels, (3) selection rules and intensities in diatomic molecules, (4) macroscopic properties of molecular gases (scattering, dispersion, Kerr and Faraday effects, dielectric constants, susceptibilities, and specific heats), and (5) molecule formation and chemical binding.

The book is written in uncommonly terse and vigorous English. Its bias is definitely theoretical, experimental details being reduced in the text to the barest minimum. There are, however, copious references to original papers (352 in all) which may be consulted for the relevant experimental data. The bibliography is carried to the end of 1929.

Dr Kronig's book will be welcomed by all who are interested in the theoretical aspects of the problems of molecular structure. The Cambridge University Press deserves a word of commendation for its production, and in particular for the successful handling of a notation abounding in complex suffixes.

H. R. R.

Annual Report of the Smithsonian Institution, 1929. Pp. xiii + 622; 59 Figs. and 91 Plates. (Washington: United States Government Printing Office, 1930.) Price $1.75.

To the searcher for varied and various knowledge, acquired in the pleasantest way by (metaphorically) sitting at the feet of the most illustrious expounders and popularisers of modern science, the *Annual Report of the Smithsonian Institution* is an Annual Event of the First Magnitude. The Secretary's report, wandering through some hundred and fifty pages, and interesting enough to those of us who wish to know how far-flung may be the activities of such an Institution as the Smithsonian, is usually taken as read.

It is the four hundred pages of the general appendix which supply goodly provand for the epicure. Here Sir James Jeans in some twenty pages discusses the physics of the universe; Dr Seares counts the stars; the querist who asks "what is light?" will find an answer supplied by Prof. A. H. Compton which may or may not be final; Prof. Donnan has his views on the mystery of life; and those who want to know how insects fly, how artificial cold is produced, how the sewing machine evolved, what are the newest views on Ur of the Chaldees, on climate and migrations, on heritable variations, on filtrable viruses, and on half a score of other interesting doings in the realm of science, will find short but authoritative essays in this same general appendix.

It is all very interesting, and we are grateful to find that no such atrocity as "Mammalogy" appears in this year's issue to offend our perhaps over-sensitive European ears.

A. F.